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Response Surface Methodology in Application of Optimal Manufacturing Process of Axial-Flow Fans Adopted by Modern Industries

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Abstract: Response surface methodology (RSM) is a collection of mathematical and statistical techniques that help in model building and analysis of problems in which a response (output variable) of interest is influenced by numerous factors (independent variables) with the objective of optimizing this response. It is widely used in many disciplines such as Manufacturing Industries, Engineering and Agricultural Sciences. Different types of axial flow fans are being used in manufacturing industries in cooling mechanisms where a lot of heat is produce by the machines and in semi-arid and arid areas to regulate room temperatures. Though little research has been done to ascertain the strength of axial flow fans, there was need to study the optimal specifications of fans to be manufactured by industries to produce a more efficient, strong and long lasting cooling fan. This new focus from the manufacturers represents new quality fans that significantly increase market profitability. In this study, second order response surface model was used to estimate the axial-flow fan parameters. Three experimental factors or specifications were evaluated, that is; the hole type in the fan "spyder" (blades), the barrel surface type onto which the “spyder” was placed, and the assembly method type for the two components. Central composite designs satisfying all the rotatability conditions were constructed. The D- and A- optimal criteria were used to evaluate the effectiveness of the design. Secondary data was used to obtained second order optimal model for manufacturing process of axial-flow fans adopted by industries. The partial derivatives of the model were used to determine the stationary points of the response surface. Contour plots were used to determine whether the stationary were at maximum, minimum or saddle points. R statistical program was used in analysis of the data.

Keywords: Response Surface Methodology (RSM), Central Composite Design (CCD), Axial-Flow Fan, Optimal

1. Introduction

The reliability of a fan is crucial in machine operation in manufacturing, for instance, where fans serve in material handling applications, a process stoppage is caused by a fan failure. A process will be shut down due to fan failure in industrial application. Also, fan operation is critical in maintain a prolific work environment in heating and cooling applications. Fan failure leads to a situation where worker yield and the quality of product declines. This is particularly correct for some production applications where air hygiene is essential in reducing the defects of production. Therefore, operation of fan has an important impact on plant production. The fan reliability is important since it always causes engineers in manufacturing sector to design fan systems conservatively [1]. Fan designers always reimburse for uncertainties in the manufacturing process by increasing capacity to manufactured fans, since they are concerned of being accountable for under-performing systems.

Response Surface Methodology (RSM) is a significant discipline in the statistical design and analysis of experiments [2]. It is broadly used in several disciplines including, Industrial, Clinical, Agricultural sciences, Biological, Food processing, Social, Engineering, amongst others. It is a technique in statistical analysis of experiments in which the production is thought to be determined by one or several manageable factors. The main goal of response surface methodology is to use a sequence of experimental designs to determine an optimum response [3]. Their idea was inspired by the necessity to carry experiments competently using a right choice of a design, and also to obtain the operating conditions with a set of manageable factors that lead to an optimum response.

Central composite design (CCD) involves the use of a two-level factorial (full or fractional) points combined with 2k axial points (star points), the distance from the center of the design space to a star point is $\pm\alpha$ where $|\alpha| > 1$ and the center of the design with one or additional points.

The increase in competition of fan manufacturing industries have led some industries to adopt the best manufacturing process to gain more customers in the market for a long lasting and strong cooling fan. This study use response surface methodology to define the best manufacturing process for obtaining a strong and quality manufactured fan to meet the end user selection.

2. Methodology

2.1. Central Composite Designs

The central composite design, CCD, is used to build a second order experimental model [4]. This design has three parts: the F factorial points, the $2k$ axial points (α), and n_c center runs. The F factorial points used are a 2^k factorial with levels at ±1. These points are used mainly for the approximation of the linear terms and the two-way interactions effects. The axial points (star points) are at a distance of α from the center of the design. They primarily contribute to the approximation of the quadratic terms. The center runs are positioned in the design space center.

Central composite design has found several applications for optimization of chemical processes such as textile dye degradation [5] and removal of the azo dye [6]. Oil agglomeration of talc was also evaluated using central composite design [7].

The axial point α is obtained as:

$$\alpha = \sqrt[4]{F} \tag{1}$$

where, F is the number of factorial points (F $= 2^k$ if it is a full factorial).

Rotatability is a very important property of the CCD [8]. It is significant to select a design that offers the same precision of estimation in all directions within a given radius [9]. It is also important to note that rotatability is achieved by using α as in (1).

2.2. Center Runs for the Rotatable CCD

The use of center runs does provide reasonable stability of $\text{SPV}(x)$ in the design region; as a result, some center runs for a rotatable CCD are very beneficial. The choice of the number of center runs provides flexibility to get a better estimate of the pure error and better power for test. Moreover, the choice of the number of center runs affects the distribution of the SPV [10].

2.3. Response Surface Model

RSM is a statistical procedure where the form of the actual association between the dependent variable y and independent variable(s) x_i , is unknown but could be estimated using low-order polynomial. In this case, the first order model is used in approximation,

$$y = \beta_0 + \sum_{i}^{k} \beta_i x_i + e \tag{2}$$

where $x_i^{'}s$ are the input variables, β_0 is an intercept and β_i are the linear regression coefficients, e is the error term and k is the number of experimental factors.

If there is curvature, a higher degree polynomial, like the second-order model is used in approximation,

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_{ii}^2 + \sum_{i<j}\sum \beta_{ij} x_i x_j + e \tag{3}$$

where y is the measured response, β_{ij} are the cross-product coefficients, β_{ii} are the quadratic terms coefficients and e is the error term with mean zero and constant variance σ^2 . The stated model (6) is utilized by the Central composite design. In matrix notation equation (6) is given by

$$\hat{y} = b_0 + x'\hat{b} + x'\hat{B}x \tag{4}$$

where b_0, $\hat{b}$ and $\hat{B}$ are estimates of the intercept, linear and second order coefficients respectively.

$$\hat{B} = \begin{pmatrix} b_{11} & \frac{b_{12}}{2} & \cdots & \frac{b_{1p}}{2} \\ \frac{b_{21}}{2} & b_{22} & \cdots & \frac{b_{2p}}{2} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ \frac{b_{p2}}{2} & \frac{b_{p2}}{2} & \cdots & b_{pp} \end{pmatrix} \tag{5}$$

Most industrial experiments use second-order response surface designs due to its desirable properties.

However, several design criteria and characteristics could be considered in selecting a second-order response surface design [11, 12]. Among the several second-order response surface designs with their distinctive characteristics is the Central Composite Design (CCD), it is the well-known and suitable in response surface exploration. The symmetry and flexibility of the design gives a considerable advantage in prediction abilities and parameter approximation. The CCD exists for spherical and cuboidal regions for two or more factors.

2.4. Location of Stationary Points

In this case, we obtained the levels of $x_1, x_2, \dots, x_k$ that optimize the predicted response. The partial derivatives with respect to each set of points $x_1, x_2, \dots, x_k$ and equating to zero were obtained as;

$$\frac{\partial \hat{y}}{\partial x_1} = \frac{\partial \hat{y}}{\partial x_2} =, \dots, = \frac{\partial \hat{y}}{\partial x_k} = 0. \quad (6)$$

Hence, points $x_1, x_2, \dots, x_k$ are called stationary points.

Its matrix notation is

$$\frac{\partial \hat{y}}{\partial x} = \hat{\boldsymbol{b}} + 2\hat{\boldsymbol{B}}x \quad (7)$$

By equating the derivative to zero, the stationary point of the system can be solved by:

$$x_s = -\frac{1}{2}\hat{\mathbf{B}}^{-1}\hat{\boldsymbol{b}} \quad (8)$$

The stationary point could represent a point of maximum, minimum and saddle points.

Contours plots were used to located these stationary points.

2.5. Factors Under Study and Their Levels

Table 1. *Factors under study and their levels*

Factors	Levels	Signs
Type of hole shape in the fan blade (spyder) (x_1)	Hex hole	-1
	Round hole	1
The shape of the hub or barrel surface (x_2)	Knurled	-1
	Smooth	1
The type of assembly method (x_3)	Staked	-1
	Spun	1

3. Results and Discussion

3.1. Manufacturing Process of Axial-Flow Fans Adopted by Modern Industries

3.1.1. Model Estimation

The estimates for the coefficients of the second order model central composite design were obtained from equation (6) by use of R statistical software, equation (13) of the response model was obtained as follows:

$$\hat{y} = 180.23 + 2.4029x_1 + 1.8705x_2 + 30.776x_3 - 43.805x_1^2 - 44.158x_2^2 - 21.006x_3^2 - 2.2335\times10^{-14}x_1x_2 + 1.25x_1x_3 - 1.00x_2x_3 \quad (9)$$

where $\hat{y}$ is the tarque (force) required to break the fan, x_1 is the hole type, x_2 the type of barrel surface attached to the hub and x_3 is the type of assembly method.

3.1.2. Model Validity

It is important to test the fitted model if it offers a suitable estimation of the correct response surface model. The Analysis of variance (ANOVA) was used to examine the central composite design model.

(i) The Analysis of Variance

Table 2. *Analysis of Variance*

Sources of Variations	Degrees of Freedom	Sum of Squares	MSS	F	P-value
Regression	9	60487.4156	6720.8240	20.9272	0.00005357
Error	9	2890.3739	321.1527		
Total	18	63377.7895			

The model significance is determined by F and p-value, the larger the F-value and the smaller the p-value is, the more the model is significant [13]. From Table 4, the F-statistic was obtained as 20.9272 with a p-value of 0.00005357, this shows that the fitted model offers adequate estimation of the correct response surface model to predict the force.

(ii) Coefficient of Determination

In order to determine how well the estimated model fits the data, R^2 and R_A^2 values were used [14]. The coefficient of determination, R^2 and adjusted R-squared, R_A^2 were obtained as 0.9544 and 0.9088 respectively. This shows that about 95% of the variation of the response was attributable to the regression model, hence the sample data fits well to the estimated model.

(iii) Testing the Adequacy of Parameters

To test the adequacy of each parameter in the model, the student t test and p-value were used as given in the Table 5

Table 3. *T-Test.*

Coefficients	Estimate	Std. Error	T-value	P-Value	VIF
Intercept	180.23	8.0051	22.5142	0.0000	
x_1	2.4029	4.8491	0.4955	0.6321	1.0000
x_2	1.8705	4.8491	0.3857	0.7086	1.0000
x_3	30.776	4.8491	6.3467	0.0001	1.0000
x_1^2	-43.805	4.8496	-9.0326	0.0000	1.0391
x_2^2	-44.158	4.8496	-9.1055	0.0000	1.0391
x_3^2	-21.006	4.8496	-4.3315	0.0019	1.0391
x_{12}	-2.2335×10^{-14}	6.3359	0.0000	1.0000	1.0000
x_{13}	1.2500	6.3359	0.1973	0.8480	1.0000
x_{23}	-1.0000	6.3359	-0.1578	0.8781	1.0000

The p-value is less than 0.05, indicates that the model is statistically significant [15]. As presented in Table 5, it was observed that the main effect coefficient of x_3 and the quadratic coefficients of x_1^2, x_2^2 and x_3^2 with p-vales of 0.0000, 0.0000 and 0.0019 respectively were highly significant ($P < 0.01$).

Table 5 indicates that the quadratic terms of the three variables were the most significant factors in determining the optimum force of the fans. The other terms, that is, the linear coefficients of x_1 and x_2, the interactive effects coefficient of x_1x_2, x_1x_3, and x_2x_3 were found to be insignificant ($P>0.05$). Therefore, the assembly method, x_3 in this study was a significant parameter that influenced the design for manufacturing of a strong axial flow fan.

Variance Inflation Factor (VIF) is widely used to measure the degree of multi-collinearity of the i[th] independent variable with other independent variables in a regression model [16].

Table 5 also indicates that VIF of linear, interaction and quadratic terms were 1.0000, 1.0000 and 1.0391 respectively, therefore, the variance of coefficients of the model were not inflated at all.

3.1.3. Location of Stationary Points

The stationary point is the one in which the response has an optimum value. In order to locate the stationary points of the response model, from (10), the first derivatives were obtained with respect to each independent variable and equated to zero to get the corresponding values of the independent variables.

The values of the independent variables were obtained by using (11), where

$$\hat{\boldsymbol{B}} = \begin{pmatrix} -43.805 & -1.1168\times10^{-14} & 0.625 \\ -1.1168\times10^{-14} & -44.158 & -0.5 \\ 0.625 & -0.5 & -21.006 \end{pmatrix} \text{and } \hat{\boldsymbol{b}} = \begin{pmatrix} 2.4029 \\ 1.8705 \\ 30.776 \end{pmatrix} \quad (10)$$

Using equation (14) in equation (12) we obtained the stationary points as 0.03790, 0.01288 and 0.7333 for x_1, x_2 and x_3 respectively.

To show whether the optimum points 0.03790, 0.01288 and 0.7333 are at maximum, minimum or saddle point, we obtained the second derivatives as:

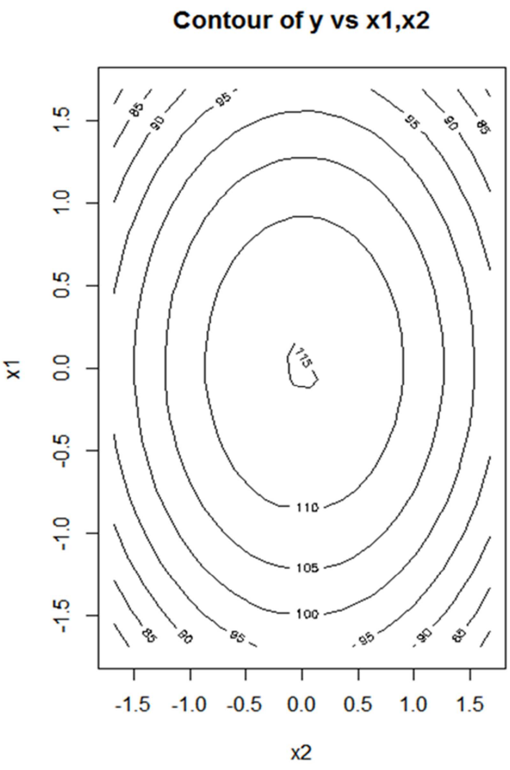
$$\frac{\partial \hat{y}}{\partial x_1} = -87.6, \ \frac{\partial \hat{y}}{\partial x_2} = -88.32, \ \frac{\partial \hat{y}}{\partial x_3} = -42.02 \quad (11)$$

Since all the second derivatives were negatives, the stationary points were at maximum point. Hence, the combination of the three factors each at high level, that is, round type of hole, smooth barrel surface and spun type of assembly method produced optimal force.

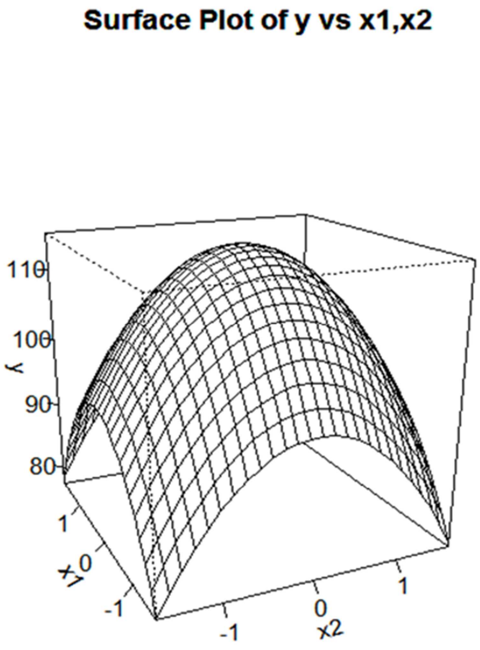
3.1.4. Contours

The graphical representation of the model in 3D response surface and 2D contour plots are used to picture out the association between the response and the levels of experiment of each factor [17]. The interactive effects of the variables and optimal levels of each variable were determined by plotting the 2D contour and 3D response surface plots. These plots are function of two factors at a time as all other factors are maintained at fixed levels, this helps in understanding both the main and the interaction effects of any two given factors.

The 2D contour and 3D response surface plots were plotted to determine the interactive effects of the variables and the optimal levels of each variable.



(a)

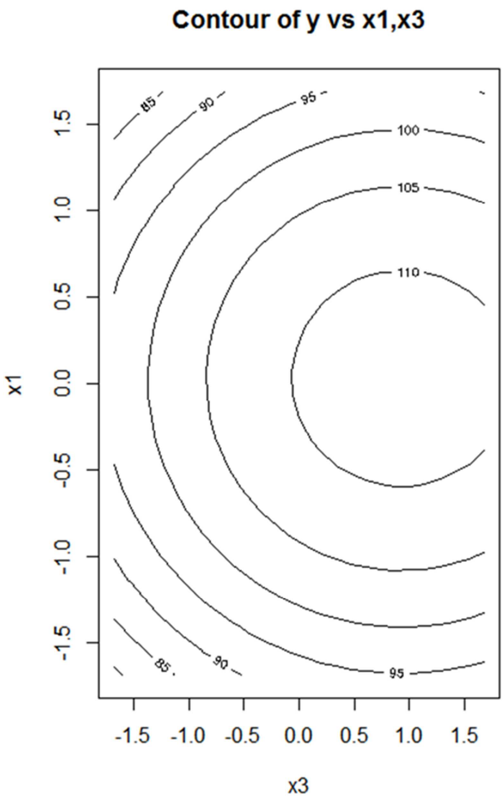


(b)

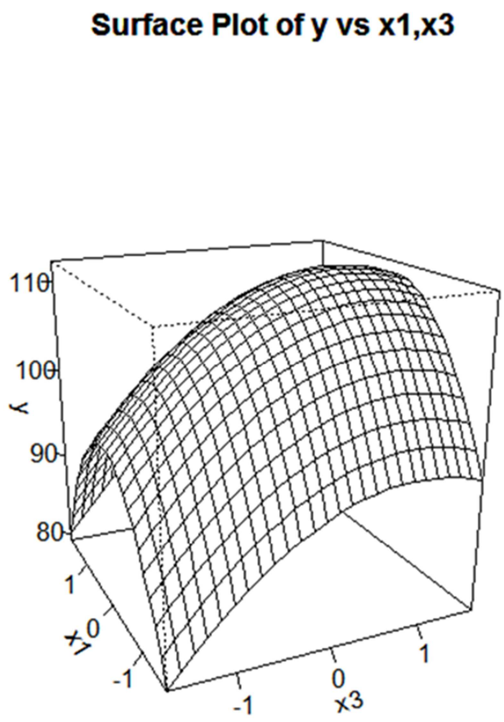
Figure 1. *(a) Contour plot (b) 3D Response surface plot.*

This shows the interaction effect of the hole type (x_1) and the shape of the hub (x_2) on torque (force) require to break the fan.

Figure 1 shows the interaction between the hole type (x_1) and the shape of the hub (x_2) on the force required to break the fan are significant. The optimum level of the interaction of the two variables exhibited that the optimal factors were precisely inside the design boundary of the response surface.



(a)

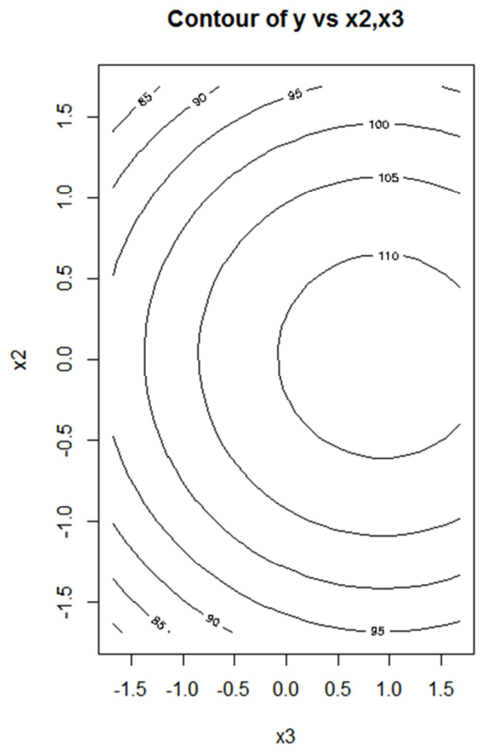


(b)

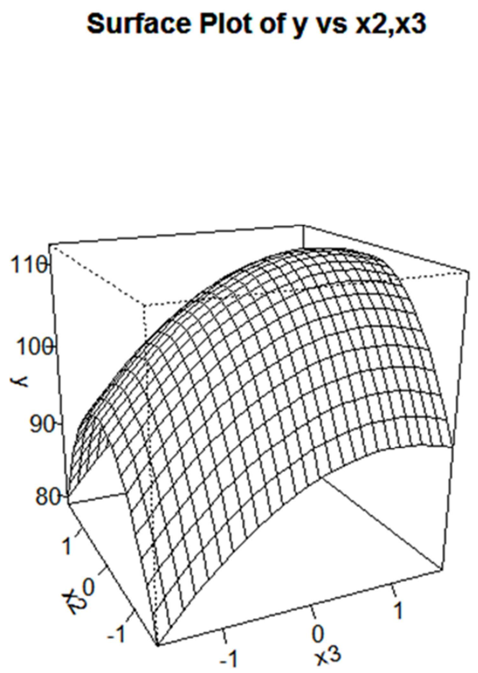
Figure 2. *(a) Contour plot (b) 3D Response surface plot.*

This shows the interaction effect of the hole type (x_1) and assembly method (x_3) on torque (force) require to break the fan.

Figure 2 shows that the interaction between the hole type (x_1) and the assembly method (x_3) on the force required to break the fan are insignificant.



(a)



(b)

Figure 3. *(a) Contour plot (b) 3D Response surface plot.*

This shows the interaction effect of the shape of the hole (x_2) and assembly method (x_3) on torque (force) require to break the fan.

Figure 3 shows that the interaction effect of the shape of the hole (x_2) and assembly method x_3 is insignificant. The interaction of each pair of the three variables resulted in the optimum level.

4. Conclusions and Recommendations

The central composite design and response surface methodology enabled the determination of optimal operating factors for the manufacturing of the axial flow fans. It was important in estimating the effect of three main independent variables (the type of hole, barrel surface and the assembly method) by using the 3D response surface and 2D contour plots. Also, a second-order polynomial model was employed to optimize the manufacturing of the fans.

The response surface model for the three factors namely; the type of hole, the barrel surface and the assembly method for the two components (type of hole and barrel surface) was found to fit the data well. The type of the assembly method for the two components and the quadratic terms of the three factors contributed significantly to the response model.

The linear effect of the assembly method type plays a critical part in the manufacturing of the axial flow fans, the quadratic terms also contributed significantly in determining the curvature for the optimal values. The linear effect of the type of hole and type of barrel surface and also the interaction of the three factors have insignificant effect on the model.

It was concluded that the stationary points were at maximum, therefore, the optimal force was obtained at high level of each factor. That is, round type of hole, smooth type of barrel surface and spun type of assembly method.

It was recommended that the type of assembly method namely; staked and spun to be considered in manufacturing the best strong and quality fan.

It was recommended that the manufacturing of a strong fan should be done using the three factors each at high level, that is, round type of hole, smooth type of barrel surface and spun type of assembly method.

It was recommended that a further research can be done on the effects of the diameter of the axial and length of the blade on the force of the axial flow-fan using natural values. Also the canonical form of the model to be obtained.

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