American Journal of Theoretical and Applied Statistics
2019; 8(1): 31-38
http://www.sciencepublishinggroup.com/j/ajtas
doi: 10.11648/j.ajtas.20190801.15
ISSN: 2326-8999 (Print); ISSN: 2326-9006 (Online)

Class of Difference Cum Ratio–Type Estimator in Double Sampling Using Two Auxiliary Variables with Some Known Population Parameters

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To cite this article:
Akingbade Toluwalase Janet, Okafor Fabian Chinemelu. Class of Difference Cum Ratio–Type Estimator in Double Sampling Using Two Auxiliary Variables with Some Known Population Parameters. *American Journal of Theoretical and Applied Statistics*. Vol. 8, No. 1, 2019, pp. 31-38. doi: 10.11648/j.ajtas.20190801.15

Received: February 2, 2019; **Accepted**: March 12, 2019; **Published**: April 1, 2019

Abstract: In this paper, a class of double sampling difference cum ratio - type estimator using two auxiliary variables was proposed for estimating the finite population mean of the variable of interest. The expression for the bias and the mean square error of the proposed estimators are derived; in addition, some members of the class of the estimator are identified. The conditions under which the proposed estimators perform better than the sample mean and the existing double sampling ratio type estimators are derived. The empirical analysis showed that the proposed class of estimator performs better than the existing estimators considered in this study.

Keywords: Mean Square Error (MSE), Ratio Estimator, Double Sampling, Percent Relative Efficiency (PRE), Auxiliary Variables

1. Introduction

Proper use of auxiliary variable is always known to improve the performance of estimators. Ratio, product and regression estimators are the most common and widely discussed in sampling theory literature. Ratio and product estimators are not as efficient as regression estimator except when the regression line passes through the origin. In real life situations, the line does not pass through the origin. This limitation has made many authors to provide alternatives to get better estimates. Authors like Kadilar and Cingi, [1, 2], Raja et al., [3], Sisodia and Dwivedi, [4], Singh and Kakran, [5], Singh and Tailor, [6], Subramani and Kumarapandiyan, [7], Upadhyaya and Singh, [8] and Yan and Tian [10] have modified the classical ratio estimator by Cochran [10]using some known population parameters like coefficient of variation, coefficient of skewness e.t.c. , of an auxiliary variable when the population mean of the auxiliary variable is known.

Sometimes it has been observed in sample surveys that information may be available on more than one auxiliary variable. Some authors like Kadilar and Cingi, [11], Mohanty, [12], Olkin, [13], Singh, [14] and Swain, [15], have worked on the use of two auxiliary variables in the estimation of the population mean of the variable interest. In their work, they assumed that the population means of the two auxiliary variables are known. In real practical survey situation, the population means of the two auxiliary variables may not be available. In this condition it is customary to use two phase sampling or double sampling scheme for estimating the population means of the auxiliary variables, see Cochran [10]. In the literature, several authorshave proposed different estimatorsin double sampling for estimating the finite population mean of the study variable using two auxiliary variables. Authors like Mohanty, [12], Mukerjee et al., [16] and Muhammad et al., [17], suggested some estimators with an assumption that the population means of the two auxiliary variables are unknown.

Mohanty suggested regression ratio estimator indouble sampling ($\hat{T}_M$) using two auxiliary variables x and z, [12]. Which is given by

$$\hat{T}_M = \left[\bar{y} + b_{yx}(\bar{x}' - \bar{x}) \right] \frac{\bar{z}'}{\bar{z}} \tag{1}$$

While Mukerjee et al, [16], suggested regression type estimator in double sampling of the form:

$$\hat{T}_{MRV} = \bar{y} + b_{yx}(\bar{x}' - \bar{x}) + b_{yz}(\bar{z}' - \bar{z}) \tag{2}$$

Muhammad et al. [17] also proposedregression type estimators by adopting Mohanty's,[12] and Mukerjee et al, [16] estimators. The estimator is given by

$$\hat{T}_{MNM} = \left[\bar{y} + b_{yx}(\bar{x}' - \bar{x})\right]\left\{\theta\frac{\bar{z}'}{\bar{z}} + (1-\theta)\frac{\bar{z}}{\bar{z}'}\right\} \tag{3}$$

where $\theta$is suitably chosen constant. $\bar{x}'$, $\bar{z}'$ are sample means based on the first phase sample; $\bar{y}, \bar{x}$, $\bar{z}$ are sample means based on the subsample.

b_{xy}(sample regression coefficient of y on x) , b_{xz} , (sample regression coefficient of x on z)

However, these authors did not consider the use of population parameters of any of the auxiliary variables like coefficient of variation, coefficient of skewness, decilese.t.c. to improve on the efficiency of the estimators. In this study, a class of difference cum ratio-type estimator in double sampling was proposed. Some known population parameters of one of the auxiliary variables were used to construct the estimator.

2. The Proposed Class of Ratio Estimator Using Two Auxiliary Variables in Double Sampling

Consider a finite population $U = \{u_1, u_2, \ldots u_N\}$ of size N. Let Y bethe study variable and$X$, Zbe the two auxiliary variables, taking values(y_i, x_i, z_i) on the i[th] unit of the population. Let $(\bar{Y}, \bar{X}, \bar{Z})$ be the population means of (y, x, z), respectively. Suppose the population means of the auxiliary variables are unknown. In such a situation we use a two phase sampling. A preliminary large sample (n')is selected using simple random sampling from N;information of the auxiliary variables are obtained from the sample. Information on the variable of interest (y) is collected from a second random sampleof size n is selected from the first phase sample ($n < n'$).

2.1. The Proposed Class of Estimator

Following Kadilar and Cingi, [1, 2] and Tripathi et al., [18], the proposed estimator is of the form:

$$\hat{T}^*_{dp} = \frac{\left[\bar{y} - t_1(\bar{x}^{\gamma} - \bar{x}'^{\gamma})\right]\left(A\bar{x}' + G\right)^{\alpha}}{\left[A\bar{x} + G - t_2(\bar{z}^{\gamma} - \bar{z}'^{\gamma})\right]^{\alpha}} \tag{4}$$

A and G are assumed known function of the auxiliary variable X such as coefficient of kurtosis ($\beta_{2(x)}$), coefficient of skewness($\beta_{1(x)}$), coefficient of variation (C_x), deciles (first decile, $D_{1(x)}$,second decile, $D_{2(x)}$, ..., tenth decile), correlation coefficient between X and Y (ρ_{xy}). Also$0 < \gamma \leq 1$,$t_1$and t_2 are unknown constants. The scalar αtakes values -1, (for product-type estimator) and + 1 (for ratio-type estimator).

2.2. Derivation of the Bias and Mean Square Error of the Proposed Estimator $\hat{T}^*_{dp}$

To obtain the Bias and MSE of $\hat{T}^*_{dp}$, up to the first order of approximation, let us define

$$\bar{x} = \bar{X}(1+\Delta_x),\ \bar{x}' = \bar{X}(1+\Delta'_x),\ \bar{x}'^{\gamma} = \left[\bar{X}(1+\Delta'_x)\right]^{\gamma},\ \bar{z}'^{\gamma} = \left[\bar{Z}(1+\Delta'_z)\right]^{\gamma}$$

$$\bar{x}^{\gamma} = \bar{X}^{\gamma}(1+\Delta_x)^{\gamma},\ \bar{z}^{\gamma} = \bar{Z}^{\gamma}(1+\Delta_z)^{\gamma},\ \bar{y} = \bar{Y}(1+\Delta_y)$$

Expressing the proposed estimator $\hat{T}^*_{dp}$ in terms of Δ'swe have

$$\hat{T}^*_{dp} = \frac{\left[\bar{Y} + \bar{Y}\Delta_Y - t_1\left[\bar{X}^{\gamma}(1+\Delta_X)^{\gamma} - \bar{X}^{\gamma}(1+\Delta'_X)^{\gamma}\right]\right]\left[A\bar{X}(1+\Delta'_X) + G\right]^{\alpha}}{\left[A\bar{X}(1+\Delta_X) + G - t_2\left[\bar{Z}^{\gamma}(1+\Delta_Z)^{\gamma} - \bar{Z}^{\gamma}(1+\Delta'_Z)^{\gamma}\right]\right]^{\alpha}} \tag{5}$$

Expanding (5) to the first order of approximation using binomial series expansion, stopping at order 2, we have

$$\begin{aligned}
\hat{T}^*_{dp} &= \bar{Y} + \bar{Y}\Delta_Y - t_1q_1\Delta_X - \frac{t_1(\gamma-1)q_1\Delta_X^2}{2} + t_1q_1\Delta'_X + \frac{t_1q_1(\gamma-1)\Delta_X'^2}{2} + \alpha\lambda\bar{Y}\Delta'_X + \alpha\lambda\bar{Y}\Delta'_X\Delta_Y \\
&+ \frac{\alpha(\alpha-1)\lambda^2\bar{Y}\Delta_X'^2}{2} - \alpha\lambda\bar{Y}\Delta_X - \alpha\lambda\bar{Y}\Delta_X\Delta_Y + t_1\alpha\lambda q_1\Delta_X^2 - t_1\alpha\lambda q_1\Delta_X'^2 - \alpha^2\lambda^2\bar{Y}\Delta_X\Delta'_X + \frac{t_2\alpha q_2\bar{Y}\Delta_Z}{K} \\
&+ \frac{t_2\alpha q_2\bar{Y}\Delta_Z\Delta_Y}{K} - \frac{t_1t_2\alpha q_1q_2\Delta_X\Delta_Z}{K} + \frac{t_1t_2\alpha q_1q_2\Delta'_X\Delta_Z}{K} + \frac{t_2\alpha^2\lambda q_2\bar{Y}\Delta'_X\Delta_Z}{K} \\
&+ \frac{t_2\alpha(\gamma-1)q_2\bar{Y}\Delta_Z^2}{2K} - \frac{t_2\alpha q_2\bar{Y}\Delta'_Z}{K} - \frac{t_2\alpha q_2\bar{Y}\Delta'_Z\Delta_Y}{K} + \frac{t_1t_2\alpha q_1q_2\Delta_X\Delta'_Z}{K} - \frac{t_1t_2\alpha q_1q_2\Delta'_X\Delta'_Z}{K} - \\
&\frac{t_2\alpha^2\lambda q_2\bar{Y}\Delta'_X\Delta'_Z}{K} - \frac{t_2\alpha(\gamma-1)q_2\bar{Y}\Delta_Z'^2}{2K} + \frac{\alpha(\alpha+1)\lambda^2\bar{Y}\Delta_X^2}{2} + \frac{\alpha(\alpha+1)t_2^2q_2^2\bar{Y}\Delta_Z^2}{2K^2} + \\
&\frac{\alpha(\alpha+1)t_2^2q_2^2\bar{Y}\Delta_Z'^2}{2K^2} - \frac{\alpha(\alpha+1)t_2\lambda q_2\bar{Y}\Delta_X\Delta_Z}{K} + \frac{\alpha(\alpha+1)t_2\lambda q_2\bar{Y}\Delta_X\Delta'_Z}{K} - \frac{\alpha(\alpha+1)t_2^2q_2^2\bar{Y}\Delta_Z'^2}{K^2}
\end{aligned} \tag{6}$$

$$q_1 = \gamma \bar{X}^{\gamma},\ q_2 = \gamma \bar{Z}^{\gamma},\ K = A\bar{X} + G,\ \ \lambda = \frac{A\bar{X}}{A\bar{X} + G}$$

Taking expectation of (6) and using the results:

$$\mathrm{E}(\Delta_y) = \mathrm{E}(\Delta_X) = \mathrm{E}(\Delta'_Z) = \mathrm{E}(\Delta'_X) = \mathrm{E}(\Delta_Z) = 0$$

$$E(\Delta_Y^2) = \omega_1 C_Y^2, E(\Delta_X^2) = \omega_1 C_X^2, E(\Delta_z^2) = \omega_1 C_Z^2\ ,\ E(\Delta_X'^2) = E(\Delta_X \Delta'_X) = \omega_2 C_X^2$$

$$E(\Delta_Z'^2) = \omega_2 C_Z^2, E(\Delta'_{XY}) = \omega_2 \rho_{xy} C_X C_Y, E(\Delta'_X \Delta_Z) = \omega_2 \rho_{XZ} C_X C_Z$$

$$E(\Delta_X \Delta_Z) = \omega_1 \rho_{XZ} C_X C_Z\ ,\ E(\Delta'_Z \Delta_Y) = \omega_2 \rho_{ZY} C_Z C_Y, E(\Delta_Z \Delta_Y) = \omega_1 \rho_{ZY} C_Z C_Y$$

$$\omega_1 = \frac{1}{n} - \frac{1}{N},\ \omega_2 = \frac{1}{n'} - \frac{1}{N},\ \omega_3 = \frac{1}{n} - \frac{1}{n'}$$

After simplification, the bias is

$$\mathrm{B}(\hat{T}_{dp}^*) = E(\hat{T}_{dp}^* - \bar{Y}) = \omega_3 \left[\begin{array}{l} -\frac{t_1(\gamma-1)q_1 C_x^2}{2} - \alpha\lambda\bar{Y}\rho_{xy}C_x C_y + \frac{\alpha(\alpha+1)\lambda^2\bar{Y}C_x^2}{2} + t_1\alpha\lambda q_1 C_x^2 + \frac{t_2\alpha q_2\bar{Y}\rho_{zy}C_z C_y}{K} - \\ \frac{t_1 t_2\alpha q_1 q_2\rho_{xz}C_x C_z}{K} + \frac{t_2\alpha(\gamma-1)q_2\bar{Y}C_z^2}{2K} + \frac{t_2^2\alpha(\alpha+1)q_2^2\bar{Y}C_z^2}{2K^2} - \frac{t_2\alpha(\alpha+1)\lambda q_2\bar{Y}\rho_{xz}C_x C_z}{K} \end{array} \right] \quad (7)$$

The mean square error of this estimator is

$$MSE(\hat{T}_{dp}^*) = E(\hat{T}_{dp}^* - \bar{Y})^2$$

Which from (6) and ignoring order higher than 2 and after simplification we have

$$\begin{aligned} \mathrm{MSE}(\hat{T}_{dp}^*) &= \omega_1\bar{Y}^2C_y^2 + \omega_3[t_1^2q_1^2C_x^2 + \alpha^2\lambda^2\bar{Y}^2C_x^2 + \frac{t_2^2\alpha^2q_2^2\bar{Y}^2C_z^2}{K^2} - 2\{t_1q_1\bar{Y}\rho_{xy}C_xC_y \\ &+\alpha\lambda\bar{Y}^2\rho_{xy}C_xC_y - \frac{t_2\alpha q_2\bar{Y}^2\rho_{zy}C_zC_y}{K} - t_1\alpha\lambda q_1\bar{Y}C_x^2 + \frac{t_1t_2\alpha q_1q_2\bar{Y}\rho_{xz}C_xC_z}{K} + \frac{t_2\alpha^2\lambda q_2\bar{Y}^2\rho_{xz}C_xC_z}{K}\}] \end{aligned} \quad (8)$$

In order to obtain the optimum values of t_1 and t_2 we differentiate (8) simultaneously with respect to t_1 and t_2 and solve the resultant equations. This gives

$$t_{1_0} = \frac{\bar{Y}C_y(\rho_{xy} - \rho_{xz}\rho_{zy}) - \alpha\lambda\bar{Y}C_x(1-\rho_{xz}^2)}{\gamma\bar{X}^{\gamma}C_x(1-\rho_{xz}^2)} = \bar{Y}\left(\frac{l_1 - \alpha\lambda}{q_1}\right)$$

$$t_{2_0} = -\frac{C_y(\rho_{zy} - \rho_{xy}\rho_{xz})\left(A\bar{X}+G\right)}{\alpha\gamma\bar{Z}^{\gamma}C_z(1-\rho_{xz}^2)} = -\frac{l_2K}{\alpha q_2}$$

Where $l_1 = \dfrac{\bar{Y}C_y(\rho_{xy} - \rho_{xz}\rho_{zy})}{C_x(1-\rho_{xz}^2)}$ and $l_2 = \dfrac{C_y(\rho_{zy} - \rho_{xy}\rho_{xz})}{C_z(1-\rho_{xz}^2)}$

Substituting the optimum values t_{1_o} and t_{2_o} in (8) we obtain the minimum mean square error

$$\mathrm{MSE}(\hat{T}_{dp}^*)_{opt} = \bar{Y}^2[\omega_1C_y^2 + \omega_3\{l_1^2C_x^2 + l_2^2C_z^2 - 2(l_1\rho_{xy}C_xC_y + l_2\rho_{zy}C_zC_y - l_1l_2\rho_{xz}C_xC_z)\}] \quad (9)$$

3. Sub-members of the Proposed Class of Ratio-type Estimator

Setting t_1, t_2, α, γ, A and G to specific values in (4), some estimators which can be regarded as members of this proposed estimator can be obtained. For instance, if $t_1 = \alpha = 0$ we have the usual sample mean estimator.

Also setting $t_1 = b_{xy} = \frac{S_{xy}}{S_x^2}$ (sample regression coefficient of y on x) , $t_2 = b_{xz} = \frac{S_{xz}}{S_z^2}$,(sample regression coeffcient of x on z), where s_x^2

and s_z^2 are thesample variances of x and z respectively, s_{xy} and s_{xz} are the sample covariances between x and y and between x and z, respectively and $\alpha = 1$ (ratio estimator), we have a member of the class of ratio-type regression estimator for estimating the population mean using two auxiliary variables when the population mean of two auxiliary variables are unknown presented below.

$$\hat{T}^*_{dp\beta} = \frac{\left[\bar{y} - b_{xy}(\bar{x}^{\gamma} - \bar{x}'^{\gamma})\right](A\bar{x}' + G)}{\left[A\bar{x} + G - b_{xz}(\bar{z}^{\gamma} - \bar{z}'^{\gamma})\right]}, 0 < \gamma \leq 1 \quad (10)$$

Using the large sample approximation as used in the case of the regression estimation of the population mean, where b_{xy} tends to $\beta_{xy} = S_{xy}/S_x^2$ and b_{xz} tends to $\beta_{xz} = S_{xz}/S_z^2$, the bias and mean square error, from (7) and (8) are as follows:

$$\begin{aligned}\mathrm{B}(\hat{T}^*_{dp\beta}) = \omega_3[&-\frac{(\gamma-1)q_1R_1\bar{Y}\rho_{xy}C_xC_y}{2} - \lambda\bar{Y}\rho_{xy}C_xC_y + \lambda^2\bar{Y}C_x^2 + \lambda q_1R_1\rho_{xy}C_xC_y + \frac{q_2R_2\bar{Y}\rho_{xz}\rho_{zy}C_xC_y}{K} \\ &-\frac{q_1q_2R_2\bar{Y}\rho_{xy}\rho_{xz}^2C_xC_y}{K} + \frac{(\gamma-1)q_2R_2\bar{Y}\rho_{xz}C_xC_z}{2K} + \frac{q_2^2R_2^2\bar{Y}\rho_{xz}^2C_x^2}{K^2} - \frac{2\lambda q_2R_2\bar{Y}\rho_{xz}^2C_x^2}{K}]\end{aligned} \quad (11)$$

$$\begin{aligned}\mathrm{MSE}(\hat{T}^*_{dp\beta}) = \omega_1\bar{Y}^2C_y^2 + \omega_3[&R_1^2q_1^2\rho_{xy}^2C_y^2 + \lambda^2\bar{Y}^2C_x^2 + \frac{R_2^2q_2^2\bar{Y}^2\rho_{xz}^2C_z^2}{K^2} - 2\{R_1q_1\bar{Y}\rho_{xy}^2C_y^2 \\ &+\lambda\bar{Y}^2\rho_{xy}C_xC_y - \frac{R_2q_2\bar{Y}^2\rho_{xz}\rho_{zy}C_xC_y}{K} - R_1\lambda q_1\bar{Y}\rho_{xy}C_xC_y + \frac{R_1R_2q_1q_2\bar{Y}\rho_{xy}\rho_{xz}C_xC_y}{K} + \frac{R_2\lambda q_2\bar{Y}^2\rho_{xz}^2C_x^2}{K}\}]\end{aligned} \quad (12)$$

Where $R_1 = \bar{Y}/\bar{X}$, $R_2 = \bar{X}/\bar{Z}$. If weset $R_1q_1 = q_3$ and $q_4 = R_2q_2/K$, equation (12) becomes

$$\begin{aligned}\mathrm{MSE}(\hat{T}^*_{dp\beta}) = \omega_1\bar{Y}^2C_y^2 + \omega_3[&q_3^2\rho_{xy}^2C_y^2 + \lambda^2\bar{Y}^2C_x^2 + q_4^2\bar{Y}^2\rho_{xz}^2C_z^2 - 2\{q_3\bar{Y}\rho_{xy}^2C_y^2 \\ &+\lambda\bar{Y}^2\rho_{xy}C_xC_y - q_4\bar{Y}^2\rho_{xz}\rho_{zy}C_xC_y - \lambda q_3\bar{Y}\rho_{xy}C_xC_y + q_3q_4\bar{Y}\rho_{xy}\rho_{xz}C_xC_y + \lambda q_4\bar{Y}^2\rho_{xz}^2C_x^2\}]\end{aligned} \quad (13)$$

Estimators in this class of ratio type regression estimator are found in Table 1.

4. Theoretical Comparison of the Proposed Estimator with Other Existing Estimators Discussed

In this section, the performance of the proposed estimator with other existing estimators were compared, through their mean square errors, like the sample mean, $\hat{T}_0 = \bar{y}$, with variance $V(\hat{T}_0) = \omega_1\bar{Y}^2C_y^2$, the estimator $\hat{T}_M$ by Mohanty, [12] found in (1); the estimators $\hat{T}_{MRV}$ by Mukerjee, [16] found in (2) and the estimator $\hat{T}_{MNM}$ by Muhammad[17] found in (3).

The mean square errors of these existing estimators are:

$$MSE(\hat{T}_M) = \bar{Y}^2\{\omega_1C_y^2 + \omega_3(\rho_{xz}^2C_z^2 - (\rho_{xy}C_y - \rho_{xz}C_z)^2 + (C_z - \rho_{yz}C_y)^2 - C_y^2\rho_{yz}^2\} \quad (14)$$

$$MSE(\hat{T}_{MRV}) = \bar{Y}^2C_y^2\{\omega_1 - \omega_3(\rho_{xy}^2 + \rho_{yz}^2 - 2\rho_{xy}\rho_{xz}\rho_{yz})\} \quad (15)$$

and

$$MSE(\hat{T}_{MNM}) = \bar{Y}^2C_y^2\{\omega_1 - \omega_3(\rho_{xy}^2 + (\rho_{yz} - \rho_{xy}\rho_{xz})^2)\} \quad (16)$$

The performance of the proposed estimator using the minimum MSE in (9) and MSE of the existing estimators presented in (14), (15) and (16).

The proposed estimator $\hat{T}^*_{dp}$ is better, in terms of having smaller MSE, than the sample mean if and only if from (9)

$MSE(\hat{T}^*_{dp})_{opt} \leq V(\hat{T}_o)$ iff

$$l_1^2C_x^2 + l_2^2C_z^2 \leq 2(l_1\rho_{xy}C_xC_y + l_2\rho_{zy}C_zC_y - l_1l_2\rho_{xz}C_xC_z)$$

From (9) and (14), the proposed estimator $\hat{T}^*_{dp}$ will perform better than Mohanty estimator $\hat{T}_M$, [12].

iff $MSE(\hat{T}^*_{dp})_{opt} \leq MSE(\hat{T}_M)$ this boils down to the condition

$$l_1^2C_x^2 + l_2^2C_z^2 - 2(l_1\rho_{xy}C_xC_y + l_2\rho_{zy}C_zC_y - l_1l_2\rho_{xz}C_xC_z) \le \omega_3(\rho_{xz}^2C_z^2 - (\rho_{xy}C_y - \rho_{xz}C_z)^2 + (C_z - \rho_{yz}C_y)^2 - C_y^2\rho_{yz}^2$$

The proposed estimator $\hat{T}_{dp}^*$ will be more efficient than the existing estimator $\hat{T}_{MRV}$ by Mukerjee et al., [16]. Using (9) and (15), $MSE(\hat{T}_{dp}^*)_{opt} \le MSE(\hat{T}_{MRV})$
iff

$$l_1^2C_x^2 + l_2^2C_z^2 - 2(l_1\rho_{xy}C_xC_y + l_2\rho_{zy}C_zC_y - l_1l_2\rho_{xz}C_xC_z) \le -(\rho_{xy}^2 + \rho_{yz}^2 - 2\rho_{xy}\rho_{xz}\rho_{yz})$$

From (9) and (16) the proposed estimator $\hat{T}_{dp}^*$ will be better than the existing estimator $\hat{T}_{MNM}$ by Muhammad et al., [17] iff $MSE(\hat{T}_{dp}^*)_{opt} \le MSE(\hat{T}_{MNM})$
which is equivalent to

$$l_1^2C_x^2 + l_2^2C_z^2 - 2(l_1\rho_{xy}C_xC_y + l_2\rho_{zy}C_zC_y - l_1l_2\rho_{xz}C_xC_z) \le -(\rho_{xy}^2 + (\rho_{yz} - \rho_{xy}\rho_{xz})^2)$$

Now comparing these estimators $\hat{T}_{dp}^*$ and $\hat{T}_{dp\beta}^*$ using (9) and (13), estimator $\hat{T}_{dp}^*$ will be more efficient than $\hat{T}_{dp\beta}^*$ iff

$$\begin{aligned}&\bar{Y}^2\left\{(l_1^2 + q_4\lambda\rho_{xz}^2)C_x^2 + (l_2^2 - q_4^2\rho_{xz}^2)C_z^2\right\} + 2\bar{Y}q_3\rho_{xy}^2C_y^2 \\ &\le \left(q_3\rho_{xy}C_y + \lambda\bar{Y}C_x\right)^2 - 2\rho_{xy}C_xC_y\left\{\bar{Y}^2(\lambda - l_1) + \bar{Y}q_3q_4\right\} \quad + 2\bar{Y}^2\left\{q_4\rho_{zy}C_xC_y + l_2\rho_{zy}C_zC_y - l_1l_2\rho_{xz}C_xC_z\right\}\end{aligned}$$

5. Empirical Comparison

In this section, the mean square errors, and percent relative efficiencies (PREs) of the existing and the proposed estimators with respect to the sample mean $\hat{T}_o$ were computed. The results are given in Tables 2 and 3. Real life data set by Chattefuee and Hadi,[19] and details of the data are as shown below:

Y-Per capita expenditure on education in 1975

X- Per capita income in 1973

Z − Number of residents per thousand living in urban areas in 1970

$$N = 50\ n' = 35\ n = 15\ \bar{Y} = 284.0612\ \bar{X} = 4675.12\ \bar{Z} = 657.8$$

$$\rho_{xy} = 0.60679\ \rho_{zy} = 0.31675\ \rho_{xz} = 0.61937\ S_y = 733.1407 b_{xy} = 0.058228$$

$$b_{xz} = 2.764116 b_{zy} = 0.13756\ C_y = 0.21776\ C_x = 0.13786\ C_z = 0.2204$$

$$D_{1(x)} = 3817\ D_{2(x)} = 3967\ D_{3(x)} = 4243\ D_{4(x)} = 4504\ D_{5(x)} = 4697$$

$$D_{6(x)} = 4827\ D_{7(x)} = 4989\ D_{8(x)} = 5309\ D_{9(x)} = 5560\ D_{10} = 5889$$

$$\beta_2 = -0.94843\ \beta_1 = 0.05675$$

The Percent Relative Efficiencies (PREs) of the existing estimators mentioned in (1), (2)and (3) and the jth members of the proposed estimators $\hat{T}_{dp\beta_j}^*$, j=1, 2, 3,…, 20 given in Table 1and $\hat{T}_{dp}^*$ (for minimum variance in (9)) with respect to the usual sample mean $\hat{T}_o$, is of the form;

PRE $=\frac{Var(\hat{T}_o)}{MSE(.)} * 100$where (.) $=\hat{T}_M$ or $\hat{T}_{MRV}$ or $\hat{T}_{MNM}$ or the proposed estimator.

The higher the percent relative efficiencies, the more efficient the estimator.

Table 1. *Members of the proposed class of double sampling ratio estimator using two auxiliary variables.*

S/No	Members of the proposed Class of ratio-type estimator $0 < \gamma \le 1$	A	G
1	$\hat{T}_{dp\beta1}^* = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x} - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}\bar{x}'$	1	0
2	$\hat{T}_{dp\beta2}^* = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x} + C_x - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}' + C_x)$	1	C_x

S/No	Members of the proposed Class of ratio-type estimator $0 < \gamma \leq 1$	A	G
3	$\hat{T}^*_{dp\beta 3} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x} + \beta_{2(x)} - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}' + \beta_{2(x)})$	1	$\beta_{2(x)}$
4	$\hat{T}^*_{dp\beta 4} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x}\beta_{2(x)} + C_x - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}'\beta_{2(x)} + C_x)$	$\beta_{2(x)}$	C_x
5	$\hat{T}^*_{dp\beta 5} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x}C_x + \beta_{2(x)} - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}'C_x + \beta_{2(x)})$	C_x	$\beta_{2(x)}$
6	$\hat{T}^*_{dp\beta 6} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x} + \rho_{xy} - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}' + \rho_{xy})$	1	ρ_{xy}
7	$\hat{T}^*_{dp\beta 7} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x}C_x + \rho_{xy} - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}'C_x + \rho_{xy})$	C_x	ρ_{xy}
8	$\hat{T}^*_{dp\beta 8} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x}\rho_{xy} + C_x - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}'\rho_{xy} + C_x)$	ρ_{xy}	C_x
9	$\hat{T}^*_{dp\beta 9} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x}\beta_{2(x)} + \rho_{xy} - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}'\beta_{2(x)} + \rho_{xy})$	$\beta_{2(x)}$	ρ_{xy}
10	$\hat{T}^*_{dp\beta 10} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x}\rho_{xy} + \beta_{2(x)} - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}'\rho_{xy} + \beta_{2(x)})$	ρ_{xy}	$\beta_{2(x)}$
11	$\hat{T}^*_{dp\beta 11} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x} + D_{1(x)} - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}' + D_{1(x)})$	1	$D_{1(x)}$
12	$\hat{T}^*_{dp\beta 12} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x} + D_{2(x)} - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}' + D_{2(x)})$	1	$D_{2(x)}$
13	$\hat{T}^*_{dp\beta 13} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x} + D_{3(x)} - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}' + D_{3(x)})$	1	$D_{3(x)}$
14	$\hat{T}^*_{dp\beta 14} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x} + D_{4(x)} - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}' + D_{4(x)})$	1	$D_{4(x)}$
15	$\hat{T}^*_{dp\beta 15} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x} + D_{5(x)} - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}' + D_{5(x)})$	1	$D_{5(x)}$
16	$\hat{T}^*_{dp\beta 16} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x} + D_{6(x)} - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}' + D_{6(x)})$	1	$D_{6(x)}$
17	$\hat{T}^*_{dp\beta 17} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x} + D_{7(x)} - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}' + D_{7(x)})$	1	$D_{7(x)}$
18	$\hat{T}^*_{dp\beta 18} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x} + D_{8(x)} - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}' + D_{8(x)})$	1	$D_{8(x)}$
19	$\hat{T}^*_{dp\beta 19} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x} + D_{9(x)} - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}' + D_{9(x)})$	1	$D_{9(x)}$
20	$\hat{T}^*_{dp\beta 20} = \frac{\bar{y} - b_{xy}(\bar{x}^\gamma - \bar{x}'^\gamma)}{\bar{x} + D_{10(x)} - b_{xz}(\bar{z}^\gamma - \bar{z}'^\gamma)}(\bar{x}' + D_{10(x)})$	1	$D_{10(x)}$

Table 2. *The MSE and PRE with respect to $\hat{T}_0$ of the existing and proposed estimators.*

Estimators	MSE	PRE
$\hat{T}_o$ Sample mean.	178.573	100
$\hat{T}_M$ byMohanty, [12]	291.6909	61.21548
$\hat{T}_{MRV}$ byMukerjee et al., [16]	144.9957	123.1485

Estimators	MSE	PRE
$\hat{T}_{MNM}$ by Muhammad et al., [17]	124.3938	143.5441
The proposed estimator $(\hat{T}^*_{dp})_{opt}$	124.076	143.91

Table 3. The MSE and PREwith respect to $\hat{T}_0$ of the proposed estimators.

$\hat{T}^*_{dp\beta j}$	MSEand PRE of theproposed estimators $\hat{T}^*_{dp\beta j}$ for j=1,…,20 such that $\alpha=1$, $t_1=b_{xy}$ and $t_2=b_{xz}$									
	$\gamma=0.1$		$\gamma=0.3$		$\gamma=0.5$		$\gamma=0.8$		$\gamma=1$	
	MSE	**PRE**	**MSE**	**PRE**	**MSE**	**PRE**	**MSE**	**PRE**	**MSE**	**PRE**
$\hat{T}^*_{dp\beta 1}$	125.0	142.80	124.98	142.9	124.87	143.0	124.65	143.2	154.04	115.9
$\hat{T}^*_{dp\beta 2}$	125.0	142.80	124.98	142.9	124.87	143.0	124.65	143.2	154.04	115.9
$\hat{T}^*_{dp\beta 3}$	125.0	142.80	124.98	142.9	124.87	143.0	124.65	143.3	154.02	115.9
$\hat{T}^*_{dp\beta 4}$	125.0	142.80	125.04	142.8	125.23	142.6	131.51	135.8	262.89	67.92
$\hat{T}^*_{dp\beta 5}$	124.9	142.90	124.81	143.1	124.23	143.7	160.05	111.6	1000.3	17.85
$\hat{T}^*_{dp\beta 6}$	125.0	142.80	124.98	142.9	124.87	143.0	124.65	143.2	154.05	115.9
$\hat{T}^*_{dp\beta 7}$	124.9	142.90	124.82	143.1	124.22	143.7	159.71	111.8	994.92	17.95
$\hat{T}^*_{dp\beta 8}$	125.0	142.80	124.96	142.9	124.78	143.1	124.38	143.6	159.25	112.1
$\hat{T}^*_{dp\beta 9}$	125.0	142.80	125.04	142.8	125.23	142.6	131.50	135.8	262.88	67.92
$\hat{T}^*_{dp\beta 10}$	125.0	142.90	124.96	142.9	124.77	143.1	124.37	143.6	159.25	112.1
$\hat{T}^*_{dp\beta 11}$	167.8	106.40	167.83	106.4	168.05	106.3	177.26	100.7	275.50	64.81
$\hat{T}^*_{dp\beta 12}$	171.1	104.40	171.1	104.4	171.34	104.2	180.99	98.66	282.20	63.28
$\hat{T}^*_{dp\beta 13}$	177.4	100.60	177.44	100.6	177.71	100.5	188.17	94.89	294.85	60.56
$\hat{T}^*_{dp\beta 14}$	183.8	97.15	183.81	97.14	184.11	96.99	195.35	91.41	307.21	58.12
$\hat{T}^*_{dp\beta 15}$	188.7	94.61	188.75	94.6	189.08	94.44	200.89	88.89	316.60	56.40
$\hat{T}^*_{dp\beta 16}$	192.1	92.91	192.19	92.91	192.53	92.74	204.73	87.22	323.04	55.27
$\hat{T}^*_{dp\beta 17}$	196.5	90.83	196.61	90.82	196.97	90.65	209.66	85.17	331.21	53.91
$\hat{T}^*_{dp\beta 18}$	205.7	86.80	205.74	86.79	206.14	86.62	219.80	81.24	347.77	51.34
$\hat{T}^*_{dp\beta 19}$	213.2	83.73	213.29	83.72	213.72	83.55	228.14	78.27	361.15	49.44
$\hat{T}^*_{dp\beta 20}$	223.6	79.83	223.69	79.82	224.17	79.65	239.59	74.53	379.23	47.08
$(\hat{T}^*_{dp})_{opt}$	MSE =124.076 PRE=143.91 at t_1 and t_2 optimum values for all j's									

6. Results and Discussion

6.1. Table 2 Results

The existing estimators $\hat{T}_{MRV}$ by Mukerjee et al.,[16] and estimator $\hat{T}_{MNM}$ by Muhammad et al.,[17]for estimating the population mean of the study variable have significant improvement on the sample mean because they have smaller MSE and higher percent relative efficiency. The proposed estimator $(\hat{T}^*_{dp})_{opt}$ is the most efficient estimator.

6.2. Table 3 Results

All the proposed estimators $\hat{T}^*_{dp\beta j}$, j=1,…,13 at $\gamma=0.1-0.5$, are more efficient than the sample mean because they have smaller MSE and higher percent relative efficiency while these proposed estimators $\hat{T}^*_{dp\beta j}$, j=14,…,20at $\gamma=0.1-1$ have no significant improvement on the sample mean because they have higher MSE and lower percent relative efficiency. The proposed estimators $\hat{T}^*_{dp\beta j}$, j=1,…,10 at $\gamma=0.1-0.5$ are the most efficient estimators because they perform better than the sample mean, the existing estimators, $\hat{T}_M$ by Mohanty [12] and estimator, $\hat{T}_{MNM}$ by Muhammad et al.,[17]. The proposed estimators $\hat{T}^*_{dp\beta j}$, j=1,…,10at $\gamma=0.1-0.5$ are estimators that utilized known parameter such as coefficient of variation, coefficient of kurtosis, coefficient of skewness of an auxiliary variable X and correlation coefficient of X and Y. The proposed estimators $\hat{T}^*_{dp\beta j}$, j=13,…,20 at $\gamma=0.1-1$, are estimators that utilize the third to tenth deciles and they do not have significant improvement on the existing estimators.

In general, from Table 3 results, The proposed class of ratio type estimator $\hat{T}^{*}_{dp}$, at t_1 and t_2 optimum values, is the most efficient estimator because it has the least MSE and the highest PRE and it slightly perform better than the existing estimators, $\hat{T}_{MNM}$ by Muhammad et al., [17]. Alternatively, a good guess of γ for the sub-members, $\hat{T}^{*}_{dp\beta j}$, j=1,…,10, when $t_1 = b_{xy}$ and $t_2 = b_{xz}$ at $\gamma = 0.1 - 0.5$ are efficient as estimator $\hat{T}^{*}_{dp\beta j}$, j=1,…,20 at t_1 and t_2 optimum values.

7. Conclusion

In this work, a class of double sampling difference cum ratio-type estimator was proposed using two auxiliary variables with known population parameters of the auxiliary variable(X). The conditions under which the proposed estimators have minimum mean square errors are mentioned in section 4. In conclusion, the proposed class of double sampling ratio type estimator $\hat{T}^{*}_{dp}$ at t_1 and t_2 optimum values and sub-members of the proposed class of estimator $\hat{T}^{*}_{dp\beta j}$, j=1,…,10 at $\gamma = 0.1 - 0.5$ performs better than the existing estimators by Mohanty, [12], Mukerjee et al., [16] and Muhammad et al., [17]. The proposed class of double sampling ratio type estimator is recommended for practical application.

Acknowledgements

The authors are thankful to the anonymous learned referees for their constructive comments that led to the improvement of the paper.

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