

Estimation of Finite Population Mean Using Ratio Estimator Based on Known Median of Auxiliary Variable in the Presence of Non-Response

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To cite this article:

Charles Wanyingi Nderitu, Herbert Imboga, Anthony Wanjoya. Estimation of Finite Population Mean Using Ratio Estimator Based on Known Median of Auxiliary Variable in the Presence of Non-Response. *American Journal of Theoretical and Applied Statistics*. Vol. 11, No. 2, 2022, pp. 75-82. doi: 10.11648/j.ajtas.20221102.12

Received: March 11, 2022; **Accepted:** April 1, 2022; **Published:** April 8, 2022

Abstract: The issue of non-response is a common phenomenon in sample surveys. Therefore, there is a need to develop ways of dealing with the challenge whenever it occurs. The current paper first introduces the stratification of the population as a result of the non-response. A theoretical review of the basic non response in sampling is as well explained and derived. The condition that leads to the first non-response estimator as proposed by the Hansen and Hurwitz. The resampling scheme for the non-response adjustment was described. This forms the bases for the new model which proposes a modified ratio estimator of the finite population mean in the presence of non-response when the population median of the auxiliary variable is known. The properties of the proposed estimators are derived and theoretically compared with existing ones. A theoretical efficiency comparison shows that the proposed estimator performs better than the existing ones. Further, the simulated numerical comparison shows that the Bias of the proposed estimator performs better, while its Mean squared error is competitive. Towards, the conclusion of the study we recommend further studies on the band with that balance the impact on the estimator in terms of the variance and the bias. Further, an exponential ratio form of the proposed estimator was recommended to be studied and its properties be examined.

Keywords: Auxiliary Information, Non-response, Bias, Mean Squared Error

1. Introduction

Sample Surveys are conducted under the assumption that the sampled units represent the study population. It is also assumed that is the sampled units will respond to the question [4]. However, sometimes this is never the case. Sometimes missing values occur in the sample due to non-response. According to Johnson, T. P. [6], non-response error arises under the following conditions; (i) when the concept implied by the researcher in the question is different from the respondent's feedback, these results in errors of measurement on a unit. (ii) failure to measure the same elements in the sample. This may be attributed to some respondents' failure to provide answers in human samples or locate the elements. (iii) At times, non-response errors may arise during data collection and editing; due 1 to a human error, the missing value might arise during coding and tabulation of the raw

data.

In the presence of the non-response, the study population is assumed to be stratified into two strata. The response and the non-response variable [5]. Considering a study variable Y_i of size N , $i=1,2,\dots,N$. we suppose there are two strata of the study population, the response stratum of size N_1 and the non-response stratum of size $N_2 = N - N_1$. This result into two respective population stratum means defined as; the population mean for the response stratum is characterized by;

$$\bar{Y}_1 = \frac{1}{N_1} \sum_{i=1}^{N_1} Y_i \quad (1)$$

While the population mean in the non-response stratum is defined by;

$$\bar{Y}_2 = \frac{1}{N_2} \sum_{i=1}^{N_2} Y_i \quad (2)$$

Suppose a random sample of size n is taken from the study

variable. Considering the case when the population is stratified as response and the non-response, according to [2], in the initial stages of the non-response estimation, Hansen and Hurwitz (1946) proposed a resampling scheme. Considering a simple random sampling without replacement of size n , let n_1 be the size of the respondent sub-stratum and let $n_2 = n - n_1$ be the size non-response stratum. Let r denote the size of the resample from n_2 non-respondents to be interviewed, also let $k = \frac{n_2}{r}$ For $k \geq 1$. suppose \bar{y}_r and \bar{y}_{nr}^* be the sample mean for the y character based on the n_1 and r units, respectively. Hansen and Hurwitz (1946) proposed an unbiased estimator defined by;

$$\bar{y}^* = w_1 \bar{y}_r + w_2 \bar{y}_{nr}^* \quad (3)$$

Where, $w_i = \frac{n_i}{n}$

The variance of the estimator \bar{y}^* is defined as $V(\bar{y}^*) = \left(\frac{1}{n} - \frac{1}{N}\right) S_y^2 + \frac{(k-1)N_2}{N} \left(\frac{s_{\{y^2\}}^2}{n}\right)$.

Where S_y^2 denotes the population mean square for the character y in N_1 response units and the population mean square for the y character in the N_2 non-response units are denoted by $s_{\{y^2\}}^2$.

Several researchers have contributed to the non-response's estimation. Such researchers include [3, 8, 1, 12, 14-16].

The current study considers the modification of model 3, where the median of the auxiliary information is known.

2. Proposed Estimator

Assuming the non-response in on the study variable only [11]. The suggested modification of the estimator 3 is defined as;

$$\bar{y}_{re} = \bar{y}^* \left(\frac{\bar{x}+M}{\bar{x}+M} \right) \quad (4)$$

Where M is the auxiliary population Median.

First-order approximation was considered in calculating the Bias and the mean squared error of the estimator [7]. Define some quantities,

$$\bar{y}^* = \bar{Y}(1 + e_1),$$

$$e_2 = \left(\frac{\bar{x}}{\bar{X}} - 1\right), E(e_1) = E(e_2) = 0, C_x^2 = \frac{S_x^2}{\bar{X}^2}, C_y^2 = \frac{S_y^2}{\bar{Y}^2}$$

$$E(e_1^2) = \frac{1-f}{n} \cdot C_y^2 + \frac{w_2(k-1)}{n} C_{\{y^2\}}^2$$

Where,

$$MSE(\bar{y}_r^*) = E(\bar{y}_r^* - \bar{Y})^2 = E\left(\bar{Y}^2(\Psi_p e_2)^2 - 2\Psi_p e_1 e_2 + e_1^2\right)$$

Therefore, the mean squared error of the proposed estimator is defined as;

$$MSE(\bar{y}_r^*) = \bar{Y}^2 \left[\Psi_p^2 \left(\frac{1}{n} - \frac{1}{N}\right) C_x^2 - 2\Psi_p \left(\frac{1}{n} - \frac{1}{N}\right) \rho C_x C_y + \left[\left(\frac{1}{n} - \frac{1}{N}\right) S_y^2 + \frac{k-1}{N} \frac{N_2 S_{y^2}^2}{n}\right] \right]$$

Keeping like terms her we get the mean squared error of the proposed estimator is equal to;

$$E(e_2^2) = \frac{V(\bar{x})}{\bar{X}^2} = \left(\frac{1}{n} - \frac{1}{N}\right) \frac{S_x^2}{\bar{X}^2}$$

$$E(e_1 e_2) = \frac{cov(\bar{y}^*, \bar{x})}{\bar{X}\bar{Y}} = \left(\frac{1}{n} - \frac{1}{N}\right) \frac{S_{xy}}{\bar{X}\bar{Y}}$$

$$S_{xy} = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{X})(y_i - \bar{Y})$$

$$S_x = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{X})^2$$

$$S_y = \frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{Y})^2.$$

Expressing the estimator in equation 4 in terms of the error terms we get,

$$\bar{y}_{re} = \bar{Y}(1 + e_1)(\Psi_p e_2 + 1)^{-1} \quad (5)$$

Where $\Psi_p = \frac{\bar{x}}{\bar{x}+M}$

Expanding the right-hand side of Equation 5 using Taylor's expansion up to the second-order approximation we get,

$$\bar{y}_r = \bar{Y}(1 + e_1) \left(1 - \Psi_p e_2 + (\Psi_p e_2)^2 + \dots \right)$$

$$\bar{y}_r = \bar{Y} \left[1 - \Psi_p e_2 + (\Psi_p e_2)^2 + e_1 - \Psi_p e_1 e_2 \right] \quad (6)$$

To obtain the Bias of the proposed estimator, we rearrange Equation 6.

$$(\bar{y}_r^* - \bar{Y}) = \bar{Y} \left[-\Psi_p e_2 + (\Psi_p e_2)^2 + e_1 - \Psi_p e_1 e_2 \right] \quad (7)$$

Taking expectation of Equation 7 we get the bias of the estimator

$$\begin{aligned} E(\bar{y}_r^* - \bar{Y}) &= E \left[-\Psi_p e_2 + (\Psi_p e_2)^2 + e_1 - \Psi_p e_1 e_2 \right] \\ &= \bar{Y} \left[(\Psi_p)^2 \left(\frac{1}{n} - \frac{1}{N}\right) C_x^2 - \Psi_p \left(\frac{1}{n} - \frac{1}{N}\right) \rho C_x C_y \right] \\ \text{bias}(\bar{y}_r^*) &= \bar{Y} \Psi_p \left[\frac{1}{n} - \frac{1}{N} \right] \left[\Psi_p C_x^2 - \rho C_x C_y \right] \end{aligned} \quad (8)$$

Squaring on both sides of Equation 7 to the second-order approximation and taking expectation, we get the mean square error defined as;

$$\begin{aligned} MSE(\bar{y}_r^*) &= E \left[\left(\bar{Y} \left(-\Psi_p e_2 + (\Psi_p e_2)^2 + e_1 - \Psi_p e_1 e_2 \right) \right)^2 \right] \\ &= E \left(\bar{Y}^2 \left[(\Psi_p e_2)^2 - 2\Psi_p e_1 e_2 + e_1^2 \right] \right) \end{aligned}$$

Expanding the above equation, we get

$$MSE(\bar{y}_r^*) = \bar{Y}^2 \left(\frac{1}{n} - \frac{1}{N} \right) \left[(\psi_p)^2 C_x^2 - 2\psi_p \rho C_x C_y + C_y^2 \right] + \left(\frac{(k-1)N_2 S_y^2}{Nn} \right) \quad (9)$$

3. Efficiency Comparison of Proposed Estimator

Table 1 presents some of the existing estimators with their bias and mean squared error.

Table 1. Existing estimators with their biases and mean squared errors.

Estimator	Bias	Mean squared error
$\hat{\bar{Y}}_r = \frac{\bar{y}}{\bar{x}} \bar{X} = r * \bar{X}$, Classical ratio [9]	$Bias(\hat{\bar{Y}}_r) = \frac{1-f}{n} \bar{Y} (C_x^2 - 2C_x C_y \rho)$	$MSE(\hat{\bar{Y}}_r) = \frac{1-f}{n} (\bar{Y})^2 (C_y^2 + C_x^2 - 2C_x C_y \rho)$
$\bar{y}^* = w_1 \bar{y}_1 + w_2 \bar{y}_{h2}$. Hansen and Hurwitz (1946)	0	$MSE(\bar{y}^*) = E(\bar{y}^* - \bar{Y})^2 = \frac{var(\bar{y}^*)}{\bar{Y}^2} = \left(\frac{1}{n} - \frac{1}{N} \right) C_y^2 + \frac{(k-1)N_2 S_y^2}{\bar{Y}^2 n}$
$T_R = \bar{y}^* \frac{\bar{x}}{x}$ Rao 1986	$Bias(T_R) = \bar{Y} \left[\frac{1}{n} - \frac{1}{N} \right] (C_x^2 - \rho C_x C_y)$	$MSE(T_R) = \bar{Y}^2 \left(\frac{1}{n} - \frac{1}{N} \right) [C_x^2 + C_y^2 - 2\rho C_x C_y] + (k-1) \frac{N_2}{N} \frac{S_y^2}{n}$
$\bar{y}_{er}^* = \bar{y}^* \exp \left[\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}} \right]$ The Singh and Kumar	$bias(\bar{y}_{er}^*) = \left(\frac{1}{n} - \frac{1}{N} \right) \bar{Y} \left[\frac{3C_x^2}{8} - \frac{\rho C_x C_y}{2} \right]$	$MSE(\bar{y}_{er}^*) = \left(\frac{1}{n} - \frac{1}{N} \right) (\bar{Y})^2 \left[C_y^2 + \frac{C_x^2}{4} - \rho C_y C_x \right] + \frac{(k-1)N_2}{nN} S_y^2$

Theorem 1

The proposed estimator (\bar{y}_r^*) is more efficient than the classical ratio estimator $\hat{\bar{y}}_r$ if

$$\rho \leq \frac{C_x^2 \left[1 - (\psi_p)^2 \right] - \frac{(k-1)N_2 S_y^2}{\bar{Y}^2 \left(\frac{1}{n} - \frac{1}{N} \right)}}{C_x [C_y - 2\psi_p C_y]}$$

Proof

Consider the case

$$MSE(\bar{y}_r^*) \leq MSE(\bar{y}_r)$$

Thus,

$$\bar{Y}^2 \left(\frac{1}{n} - \frac{1}{N} \right) [\psi_p^2 C_x^2 + C_y^2 - 2\psi_p C_x C_y \rho] + \frac{(k-1)N_2 S_y^2}{Nn} \leq \left(\frac{1}{n} - \frac{1}{N} \right) \bar{Y}^2 [C_x^2 + C_y^2 - 2C_y C_x \rho]$$

$$\psi_p^2 C_x^2 - 2\psi_p \rho C_x C_y + \left[\frac{(k-1)N_2 S_y^2}{\bar{Y}^2 \left(\frac{1}{n} - \frac{1}{N} \right)} \right] \leq C_x^2 - 2C_x C_y \rho$$

$$\rho [-2\psi_p C_x C_y + 2C_x C_y] \leq C_x^2 [1 - \psi_p^2] - \frac{(k-1)N_2 S_y^2}{\bar{Y}^2 \left(\frac{1}{n} - \frac{1}{N} \right)}$$

$$\rho \leq \frac{C_x^2 [1 - (\psi_p)^2] - \frac{(k-1)N_2 S_y^2}{\bar{Y}^2 \left(\frac{1}{n} - \frac{1}{N} \right)}}{C_x [C_y - 2\psi_p C_y]}$$

Theorem 2

The proposed estimator (\bar{y}_r^*) is more efficient than the Hansen and Hurwitz (1946) estimator \bar{y}^* if

$$\rho \geq - \frac{C_y^2 - \bar{Y}^2 [\psi_p^2 C_x^2 - C_y^2]}{2\psi_p C_x C_y \bar{Y}^2}$$

proof

Consider the case

$$MSE(\bar{y}_r^*) \leq MSE(\bar{y}^*)$$

Thus,

$$\bar{Y}^2 \left(\frac{1}{n} - \frac{1}{N} \right) [\psi_p^2 C_x^2 + C_y^2 - 2\psi_p C_x C_y \rho] \leq \left(\frac{1}{n} - \frac{1}{N} \right) C_y^2$$

$$\bar{Y}^2 [\psi_p^2 C_x^2 + C_y^2 - 2\psi_p C_x C_y \rho] \leq C_y^2$$

$$\rho \geq -\frac{C_y^2 - \bar{Y}^2 [\psi_p^2 C_x^2 - C_y^2]}{2\psi_p C_x C_y \bar{Y}^2}$$

Theorem 3

The proposed estimator (\bar{y}_r^*) is more efficient than the ratio-exponential estimator proposed by \bar{y}_{er}^* if \

$$\rho \leq \frac{C_x [\frac{1}{4} - \psi_p^2]}{C_y (1 - 2\psi_p)}$$

proof

Consider the case

$$MSE(\bar{y}_r^*) \leq MSE(\bar{y}_{er}^*)$$

Thus,

$$\bar{Y}^2 \left(\frac{1}{n} - \frac{1}{N} \right) [\psi_p^2 C_x^2 + C_y^2 - 2\psi_p C_x C_y \rho] \leq \bar{Y}^2 \left(\frac{1}{n} - \frac{1}{N} \right) \left[\frac{C_x^2}{4} + C_y^2 - \rho C_x C_y \right]$$

$$[\psi_p^2 C_x^2 - 2\psi_p C_x C_y \rho] \leq \left[\frac{C_x^2}{4} - \rho C_x C_y \right]$$

$$\rho \leq \frac{C_x^2 [\frac{1}{4} - \psi_p^2]}{C_x C_y (1 - 2\psi_p)}$$

$$\rho \leq \frac{C_x [\frac{1}{4} - \psi_p^2]}{C_y (1 - 2\psi_p)}$$

Theorem 4

The proposed estimator (\bar{y}_r^*) is more efficient than the ratio estimator proposed by Rao 1986, if

$$\rho \leq \frac{C_x (1 - \psi_p^2)}{2C_y (1 - \psi_p)}$$

proof

Consider the case

$$MSE(\bar{y}_r^*) \leq MSE(T_R)$$

Thus,

$$\bar{Y}^2 \left(\frac{1}{n} - \frac{1}{N} \right) [\psi_p^2 C_x^2 + C_y^2 - 2\psi_p C_x C_y \rho] \leq \bar{Y}^2 \left(\frac{1}{n} - \frac{1}{N} \right) [C_x^2 + C_y^2 - 2\rho C_x C_y]$$

$$\psi_p^2 C_x^2 - 2\psi_p C_x C_y \rho \leq C_x^2 - 2\rho C_x C_y$$

Thus,

$$\rho \leq \frac{C_x (1 - \psi_p^2)}{2C_y (1 - \psi_p)}$$

4. Data Analysis and Presentation

Two populations were used in the analysis. For the efficiency comparison of the proposed estimator, the comparison was made under various non-response rates and values of the k.

4.1. The Review of the Singh and Kumar Estimator

Singh, R et al. [13] proposed a ratio-exponential based estimator of the population mean in presence of the non-response error. The proposed estimator use population mean of the auxiliary variable to improve the performance of the Hansen and Hurwitz estimator. The proposed estimator was

found to performs better than the Hansen and Hurwitz estimator. However, as the non-response rate increases the estimator was found to be less efficient. The study concludes by stating that estimators the uses auxiliary information performs better than Hansen and Hurwitz estimator.

4.2. The Population Mean Estimates

Here we present the estimated mean under various non-response rates and the values of k for the two populations. Under all values of k, when the non-response rate is 0%, the estimated population mean is approximately equal for all estimators. However, as the non-response rate increase, the estimate changes. Comparing the changes for the proposed estimator, the changes were not extensive.

Table 2. The estimate of the population means for the first population by the non-response rate and k values.

K	Non-response percentage	Hansel & Hurwitz	New	Rao	Ratio	Sign & Kumar
1	0	1.9610	1.9610	1.9682	1.9682	1.9646
	25	1.9721	1.9610	1.9706	1.9566	1.9705
	50	1.9981	1.9728	1.99762	1.9532	1.9828
	90	2.0045	1.9815	1.9999	1.9632	1.9965
1.5	0	1.9609	1.9609	1.9660	1.9605	1.9635
	25	1.9774	1.9628	1.9740	1.9706	1.9740
	50	1.9877	1.9791	1.9886	1.9825	1.9851
	90	2.0334	2.0100	2.0180	2.0456	2.023
2	0	1.9609	1.9609	1.9607	1.9609	1.9607
	25	1.9684	1.9609	1.9700	1.9591	1.9668
	50	1.9786	1.9652	1.9719	1.9562	1.9752
	90	2.0138	1.9923	2.0286	1.9523	2.0212
3	0	1.9609	1.9607	1.9604	1.9609	1.9607
	25	1.9832	1.9801	1.9949	1.9795	1.9891
	50	1.9895	1.9804	1.9970	1.9899	1.9964
	90	2.0444	2.0347	2.0399	2.0598	2.0387

Similarly, Table 1 presents the population mean estimates for the second population. It can be observed under no response error (0%), the estimated mean is approximately the same for all the estimators. However, differences occur

as the non-response percentage increases and as k values changes from 1 through 3. It can be observed that the change in the new proposed estimator is not significant compared to others.

Table 3. The estimate of the population means for the second population by the non-response rate and k values.

k	Non-Response percentage	Hansel & Hurwitz	New	Rao	Ratio	Sign & Kumar
1	0	1.9687	1.9686	1.9698	1.9678	1.9689
	25	1.9983	1.9796	1.9999	1.9985	1.9897
	50	2.0057	2.0003	2.0056	2.0068	2.0053
	90	2.0084	2.0032	2.0076	2.0089	2.0062
1.5	0	1.9686	1.9686	1.9691	1.9601	1.9683
	25	1.9691	1.9679	1.9689	1.9697	1.9680
	50	2.0062	1.9894	2.0041	2.0064	2.0032
	90	2.0328	2.0010	2.0054	2.0364	2.0034
2	0	1.9686	1.9681	1.9745	1.9745	1.9716
	25	1.9987	1.9776	1.9892	2.0158	1.9816
	50	2.0275	2.0079	2.0100	2.0358	2.0086
	90	2.0378	2.0205	2.0332	2.0458	2.0315
3	0	1.9686	1.9671	1.9695	1.9655	1.9671
	25	1.9897	1.9772	1.9798	1.9921	1.9772
	50	2.0034	1.9893	2.0014	2.0039	1.9993
	90	2.0237	2.0015	2.0167	2.0297	2.0124

4.3. Bias

We present the Bias of the first population over the various non-response percentages and the value of K . The results are shown in Figure 1.

The graph shows the Hansel and Hurwitz estimator as an unbiased estimator; however, comparing the proposed estimator with the other existing, the resulting estimator was negligible in all the values of K . Similar, in population two,

the Hansel and Hurwitz estimator was found to be unbiased. Compared to the other estimator, the new proposed estimator was found to have resulted in small Bias under various values of k and the non-response rate. This is presented in Figure 2. Besides, it is worth noting that as the non-response rate increase, Bias also increases. Bias was also found to increase as the k values changed from 1 to 3 in all the estimators under the two populations.

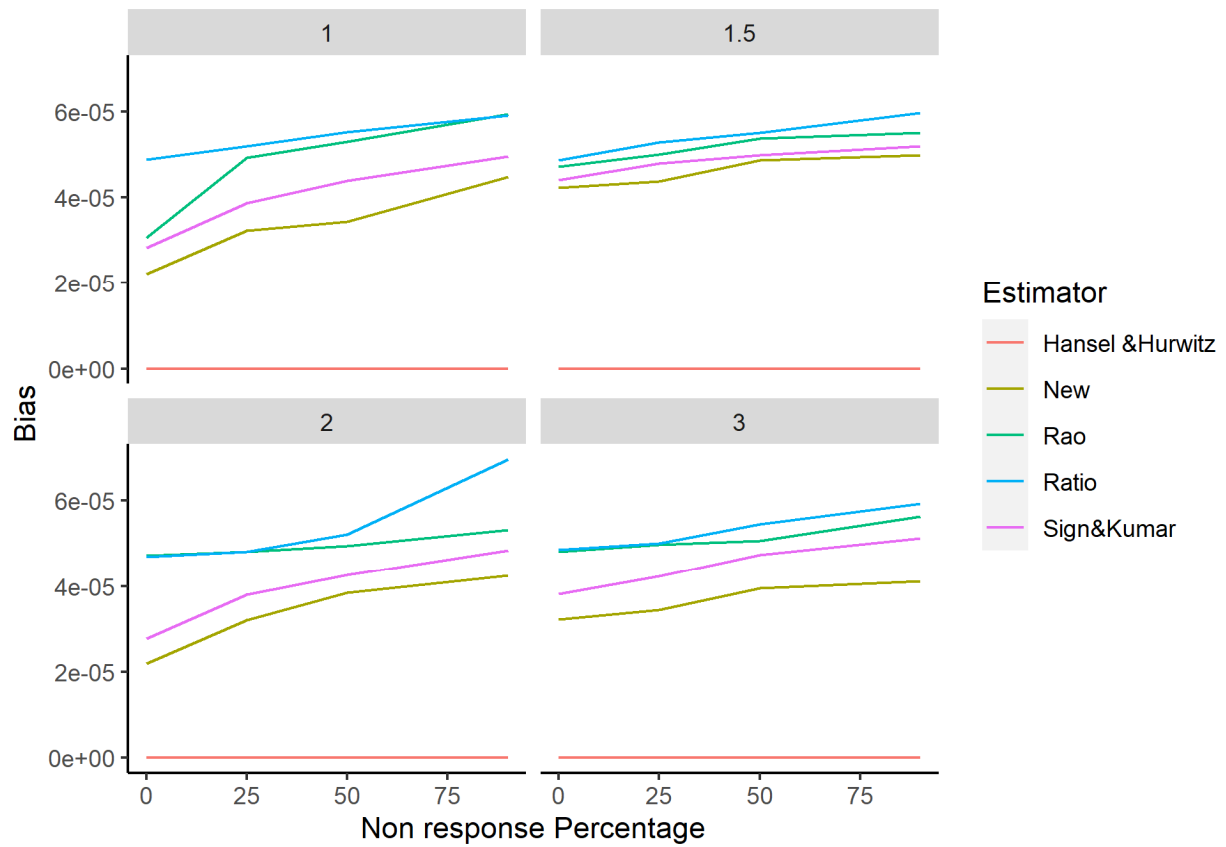


Figure 1. The Bias of the first population by non-response rates.

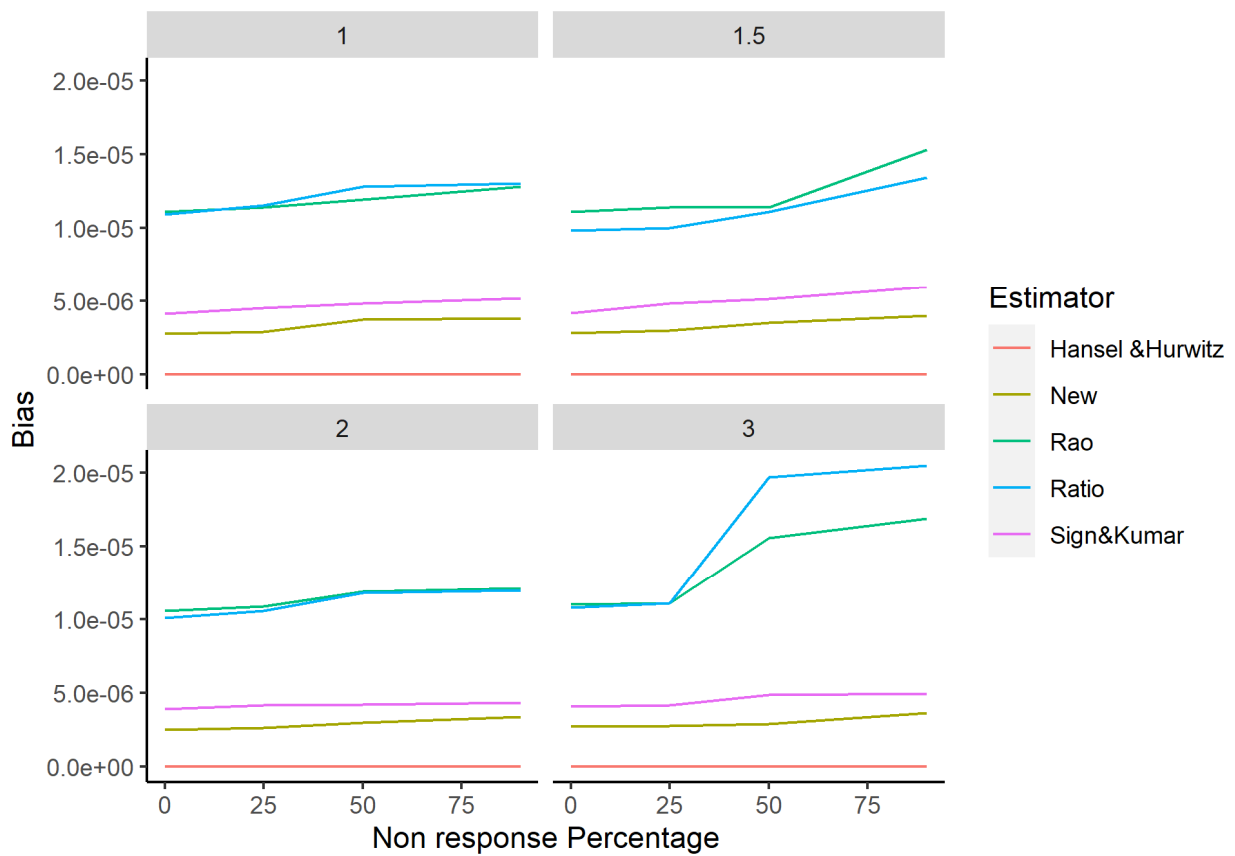


Figure 2. The Bias of the second population by non-response rates.

4.4. Mean Squared Error

The study was interested in comparing the mean squared error of the proposed estimator with the existing ones. The comparison of the mean squared error for the first population is presented in Figure 3. When the $k=1$, the curve for the

proposed estimator lies below all other estimators; when the k value is increased to 1.5, 2 and 3 still, the estimator's curve lies below. It was observed that the mean squared error would increase as non-response percentages increase and as the k value changes from 1 to 3.

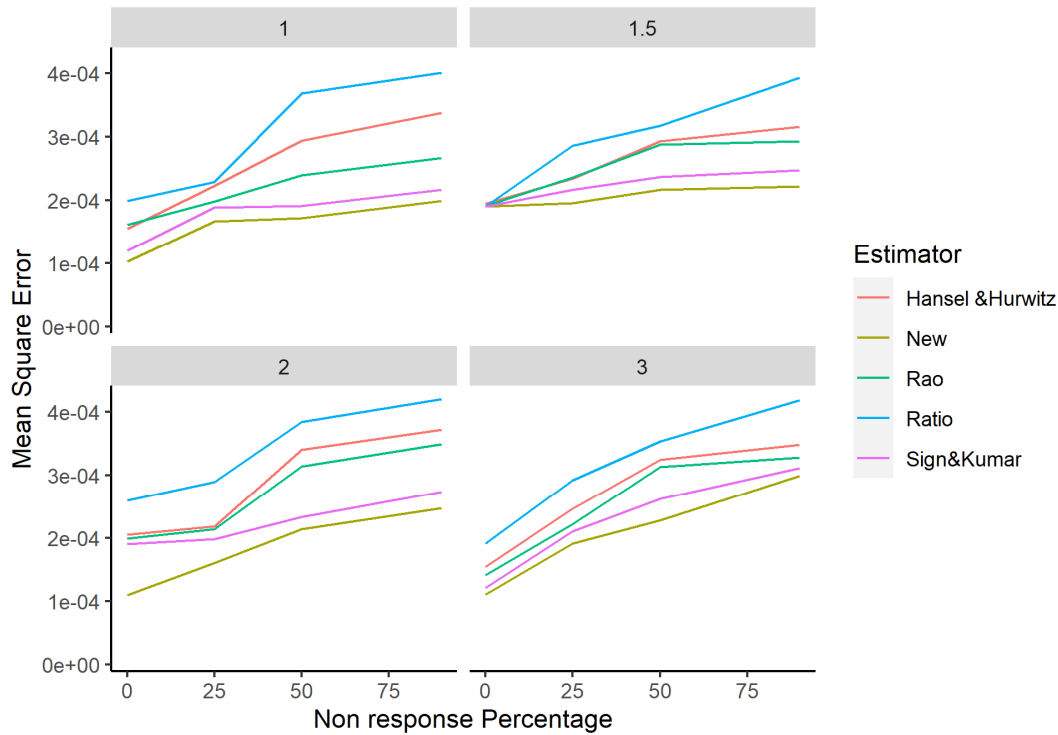


Figure 3. The Mean Squared Error for the first population by non-response percentages.

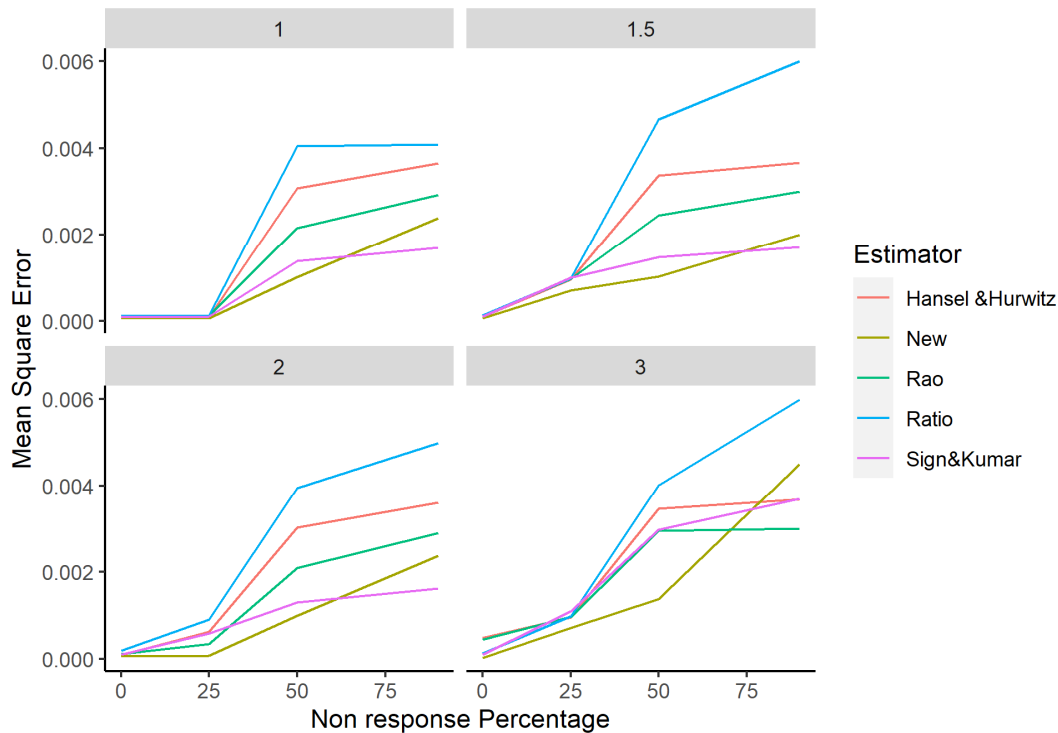


Figure 4. The Mean Squared Error for the second population by non-response percentages.

They are considering the mean squared error for the second population. As shown in Figure 4, the results suggest that as up to the 50% non-response percentage, the proposed estimator results in the slightest mean squared error. However, under severe mean squared error rates, the new estimator was not the best. This suggests that the proposed estimators were competitive in the mean squared error compared to the existing estimators.

5. Conclusion

It was noted that the Hansel and Hurwitz estimator is unbiased. However, the estimator suffers from a high mean squared error [10]. Therefore, the present study found a need to incorporate median of the auxiliary information in the development of a more efficient estimator in terms of the Mean squared error. The first observation is that the changes in the proposed estimator were small compared to other estimators. When comparing the Bias of the proposed estimator with the existing one, the results show that the proposed estimator was the smallest one. The mean squared error of the proposed estimator was found to be the smallest in most cases. However, under the severe non-response rate, the new estimator did not perform well. This suggests that applying the median in modifying the non-response estimator resulted in reduced Bias. Therefore, this remains an area to study further. It is also recommended that an exponential form of the proposed estimator be reviewed and its asymptotic properties evaluated.

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