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Comparative Analysis of Efficiency of Maximum Likelihood and Minimum Distance Estimation Techniques in Estimating Wind Distribution Parameters

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Abstract: Wind distributions are essential in making predictions on chances of getting particular wind speeds and even the ability of particular areas producing specified wind power. However, the accuracy of the parameters in predicting the wind speeds and potential wind power depends on the robustness of the distribution parameters in the fitted wind distribution model. The robustness of the parameter however depends on the estimation technique employed in the estimation of the distribution parameters. Past studies have shown that various researchers have used methods such as Maximum Likelihood Estimation (MLE), Minimum Distance Estimation (MDE) methods and other methods such as Method of Moments and Least Square Estimation technique. Despite this, the studies have not been able to compare the efficiency of the techniques estimating parameters for wind distributions to determine which of the technique is more efficient. The study aimed at determining the most efficient method in estimating the distribution parameters for wind speed using the hourly wind data for Narok County in Kenya, from January 2016 to December 2018. The study fitted both 2 parameter and 3 parameter distributions for wind in the region using the two techniques and then compared the relative efficiency of the estimated parameters. The results showed that both 2 and 3 parameter distributions fitted using the Maximum Likelihood Estimation (MLE) technique had smaller relative efficiency compare to those of Minimum Distance Estimation (MDE) technique. In conclusion, the results were able to determine that MLE gave out more efficient parameters for wind distribution than the MDE technique. The study therefore, recommended the use of MLE technique in estimating the parameters of wind distributions.

Keywords: Maximum Likelihood Estimation, Minimum Distance Estimation, Weibull, Gamma, Lognormal, AIC, BIC

1. Introduction

In statistical estimation, an estimator is considered to be more efficient when it exhibits the least variation among the other estimator of the same parameter. Statistically, the more efficient an estimator is, the more accurate it is in estimating the population parameter of interest. Therefore, a statistical estimator is considered to be more robust when it is more efficient.

The estimator technique employed in estimating parameter of interest is always known to influence the type of estimates that we obtain in a statistical estimation. As a results, the technique employed in estimating parameters is known to affect the efficiency of the estimates that we obtain. The efficiency of an estimator in statistic is known to affect accuracy of the statistical estimate in estimating the parameter of interest, as a results if the estimates are applied in real life applications, it may affect the accuracy of those application which may affect the individuals applying the technique either directly or indirectly.

Statistical estimation techniques such as Method of Moments (MOM), Maximum Likelihood Estimation (MLE), Least Square Estimation (LSE) and Minimum Distance Estimation (MDE), have been used by past researchers to fit

wind speed data to probability distributions. The fitted probability distributions have thereafter been used to estimate the chances of observing specified wind speeds in different regions and also even estimate the potential wind power of the region using the distributions. Due to the immense applications of these distributions, it is always important that one is able to obtain accurate estimates of the distribution parameters so as to guarantee that the predictions made from the distributions as accurate and reliable.

Like any other statistical estimates, an unbiased and efficient estimator is always expected to be more accurate and reliable in its predictions, therefore, when estimating the probability distribution for wind speed it is always advisable to choose efficient estimates, however, given that the estimates obtained are determined by the estimation method employed in the process of estimating the distribution parameter. Over the years, studies have been conducted to compare these estimation techniques used in estimating parameters for wind probability distributions and the have arrived at varied conclusions.

A study by Saleh, H. [20] assessed different methods used to estimate Weibull parameters namely; mean wind speed and standard deviation, maximum likelihood method, method of moments, the commonly used graphical method, modified maximum likelihood method and Power density method. The study concluded that mean wind speed and maximum likelihood estimation methods are the best methods for estimating the Weibull distribution parameters for the purpose of wind speed analysis.

A study by Johnson, W. [11] compared maximum likelihood estimation technique and method of moment technique on gamma distribution and reached a conclusion that maximum likelihood estimation is superior to the method of moments recommending that researchers should use maximum likelihood estimation technique.

Studies compared the three estimation methods; maximum likelihood estimation (MLE), method of moments (MOM) and least square estimation (LSE) method. Both articles reached a conclusion that maximum likelihood method gives good estimates compared to MOM and LSE [5, 16, 21]. It is further explained that least square method gives more accurate results compared to method of moments [5].

A study by Woodward, A. [27], which compared minimum distance and maximum likelihood techniques using mixture of asymmetric distributions concluded that minimum distance method is more robust than maximum likelihood estimation in that it is less sensitive to symmetric departures from the existing normality assumption of component distributions.

A study by Sultan, M. [23] applied minimum distance method and maximum likelihood method to estimate 3-parameters for Weibull distribution and concluded that minimum distance methods showed significant improvements over the maximum likelihood estimation method meaning that it is superior [23].

Another study by Mumford, A. [15] conducted a robust parameter estimation for mixed Weibull distribution with seven parameters using minimum distance estimation and maximum likelihood estimation. The study came to a conclusion that minimum distance estimation gives better estimates compared to maximum likelihood estimation technique.

Lastly, it was found out that MDE technique was a promising alternative to maximum likelihood estimation techniques, because it was less sensitive to the four-problematic assumption of maximum likelihood estimation. Thus, it is referred to as a robust estimation technique because it attempted to protect against minor deviations from the underlying assumptions of MLE [15, 27].

In conclusion, past studies have given varied conclusion with regard to the best estimation method for wind speed distribution when it comes to Maximum likelihood and Minimum distance estimation techniques. However, the studies have established that the two methods are superior in estimation of parameters for wind speed distributions compared to other methods such as least square estimation method, mean method and graphical method. However, none of the studies have been able to compare two superior methods of estimation (MLE and MDE) on the basis of efficiency in estimates obtained. Past literature has also shows that MDE is a promising alternative to maximum likelihood estimation because it is not too strict during estimation when the assumptions are violated as compared to MLE technique a factor that makes the estimates more robust than MLE especially when the assumptions are violated. The study aimed at contributing further to the debate by comparing the efficiency of MLE and MDE in estimation of parameters for probability distribution of wind speed, this would assist in determining the most efficient methods among the two in terms of efficiency of the estimates and also it will give an alternative mean of comparing the two techniques in order to determine the best among them.

2. Method

2.1. Data

To attain the study objective, hourly wind speed data collected for a period of three years (2016 – 2018) from five sites within Narok county namely: Irbaan primary, Imortott primary, Mara conservancy, Oldrkesi and Maasai Mara University was used. The study used the data which comprised of over 60000 observations. The study fitted the collected data into various known probability distributions that are known describe the distribution of wind speed. The probability distributions that were fitted in the study were Weibull, Gamma and Lognormal Distribution. The study fitted both two and three parameter distributions for the three mentioned distributions.

2.2. Distributions Used for Studying the Data

This study used fitted the following two and three parameter distributions for the wind data set.

2.2.1. Two Parameter Distributions

i. Weibull Distribution

Researchers who have used Weibull distribution to analyze wind speed concluded that the Weibull distribution function is the best in estimating the parameters of wind speed. The Weibull distribution model applied by the researcher is given by [2, 13, 26];

$$f(u) = \frac{b}{p}\left(\frac{u}{p}\right)^{b-1} \exp\left[-\left(\frac{u}{p}\right)\right]^{b}, (b, u > 0, p > 1), \quad (1)$$

where:

f(u) is the probability of observing wind speed,

u is the wind speed,

b is the shape factor (parameter) which has no unit but range from 1.5 to 3.0 for most wind conditions,

p is the value in the unit of wind speed called the Weibull scale parameter in m/s.

ii. Lognormal Distribution

Wind speed analysis is very wide and one of the statistical distributions used in examining the wind data is the log-normal statistical model with parameters v and k [3, 25]. The log-normal density function with the two parameters is given by:

$$f(p) = \frac{1}{k\sqrt{2p\pi}} \exp\frac{\left(\ln p - v\right)^2}{2k^2}, \quad (2)$$

where:

p is the log-normal random variable,

ln(p) is the normal random variable,

v is the mean for a normal random variable,

k is the standard deviation for the normal random variable.

iii. Gamma Distribution

The probability density function of gamma random variable y in combination with two parameters z and q representing the shape and scale parameters respectively is given by [4, 10].

$$f(y, z, q) = \frac{y^{z-1} \exp\left(-\frac{y}{q}\right)}{\Gamma(z)q^{z}}, (z, y, q, > 0) \quad (3)$$

where:

$$\Gamma(v) = \int_{0}^{\infty} y^{v-1} \exp^{-y} dy, v > 0$$

And:

z is the shape parameter,

q is the scale parameter,

y are the random variables (wind speed).

2.2.2. Three Parameter Distributions

i. Weibull Distribution

The Weibull statistical distribution with three parameters is given by [2, 3, 5, 24];

$$f(u) = \left(\frac{b}{p}\right)\left(\frac{u-w}{p}\right)^{b-1} exp\left[-\left(\frac{u-w}{p}\right)\right]^{b} \ (b, u > 0, p > 1) \ (4)$$

Where:

u is the wind speed

b is the shape parameter

p is the scale parameter measured in m/s

w is the thresh-hold parameter

ii. Lognormal Distribution with 3 Parameters

This distribution has three parameters namely scale parameter, shape parameter and thresh-hold parameter also known as location parameter. The probability density function and the cumulative density function are given by the below equations [4, 17, 25],

$$f(p) = \frac{1}{(p-y)\, k\sqrt{2\pi}} \, exp\left[-\left(\frac{\ln(p-y)-v}{2k}\right)\right]^{2} \quad (5)$$

Where:

v > 0 is the scale parameter.

k > 0 is the shape parameter.

y is the thresh-hold parameter, also referred to the location parameter.

p ≥ is the wind speed

iii. Gamma Distribution with 3 Parameters

From the past studies [11, 14, 26], the gamma function is given as follows;

$$f(y, z, q, t) = \frac{yt^{z-1}}{\Gamma(z/t)q^{z}} \, exp\left[-\left(\frac{y}{q}\right)^{t}\right] \ (z, y, q, t > 0) \ (6)$$

Where:

q is the scale parameter.

z, is shape parameters

t is the thresh-hold parameter

The Γ is defined by

$$\Gamma(v) = \int_{0}^{\infty} y^{v-1} \, exp^{-y} dy, v > 0$$

2.3. Estimation Methods

The study used both the Maximum Likelihood Estimation technique and the Minimum Distance Estimation technique to estimate. In the process of estimation, each of the estimation technique was used as follows in estimating the parameters of the distributions.

2.3.1. Maximum Likelihood Estimation Method (MLE)

According to Zhang, S. [28], maximum likelihood method can be applied in many problems since it has a strong intuitive appeal and it yield a precise estimator. He also stated that the maximum likelihood method is widely used because it is more precise especially when dealing with large sample size since it yields accurate estimator for such samples.

According to [10], maximum likelihood let's say $\hat{M}$ of M is a solution to the maximization problem given as

$$\hat{M} = argMax\, Ln(M: x1, x2, \ldots \ldots, xN) \qquad (7)$$

Where X1,...., XN represents the wind speed observations. Under suitable regularity conditions, the first order condition is given as

$$\frac{\partial Ln(M: x1, \ldots, xN)}{\partial M} = -N + \frac{1}{M}\left(\sum_{i=1}^{N} xi\right) \qquad (8)$$

MLE is a recommended technique for many distributions because it uses the values of the distribution parameters that makes the data more likely than any other parameters. This is achieved by maximizing the likelihood function of the parameters given the data. Some good features of maximum likelihood estimators is that they are asymptotically unbiased since the bias tends to zero as the sample size increases and also they are asymptotically efficient since they achieve the Cramer-Rao lower bound as sample size approaches ∞ and lastly they are asymptotically normal [8, 10].

For the two parameter distributions, the shape parameter is dimensionless and shows how the wind speed of site under examination peaked and the scale parameter is to show how windy the site under examination is (spread of the wind speed). Increasing the variation/spread of the wind speed (scale parameter) reduces the peak of the site (shape parameter) and vice versa.

i. MLE for Weibull Distribution with 2-P

This study used the Weibull two parameter distribution for the wind speed analysis which is given as [3].

$$f(u) = \left(\frac{b}{p}\right)\left(\frac{u}{p}\right)^{b-1} \exp\left[-\left(\frac{u}{p}\right)\right]^{b}, (b, u > 0: p > 1) \quad (11)$$

According to [5], the two constants, shape and scale parameters are positive constants, the scale parameter is scale to the u variable (wind speed variable) and the shape parameter decides shape of the rate function;

$$1 - f(u) = \left(\frac{b}{p}\right)\left(\frac{u}{p}\right)^{b-1}$$

If the shape parameter b, is less than 1, then the rate is decreasing with u. Whereas if shape parameter is greater than 1, then the rate is increasing with u and if the shape parameter = 1, then the rate is said to be constant and in this case the Weibull distribution is said to be the exponential distribution.

These conditions are generally called the likelihood or log-likelihood equations. The first derivative or gradient of a condition (log-likelihood) solved at point $\hat{M}$ satisfies the following equation

$$\frac{\partial Ln(M: x1, \ldots, xN)}{\partial M} = \frac{\partial Ln\left(\hat{M}: x1, \ldots, xN\right)}{\partial M} = 0 \quad (9)$$

The log-likelihood equation that corresponds to linear or non-linear system of P equations with P unknown parameters M1, M2,... MP is given by;

$$\frac{\partial Ln(M: x1, \ldots, xN)}{\partial M} = \left(\frac{\partial Ln(M: x1, \ldots, xN)}{\partial M1}, \ldots, \frac{\partial Ln(M: x1, \ldots, xN)}{\partial Mp}\right) = (0, \ldots, 0) \qquad (10)$$

Suppose that $u_1, u_2, \ldots \ldots, u_n$ are independent and identically distributed Weibull random variables representing the wind speed with a probability density function f(u) given in the equation (11) where the two parameters are assumed to be unknown. To estimate the parameters using maximum likelihood method, the likelihood function of $u_1, u_2, \ldots \ldots, u_n$ can be formulated from equation (11) as shown in equation (12).

The product of the constants are not performed (introducing the general summation including the constants is not necessary to avoid interference with the rate function in the Weibull distribution). The aim is to understand how the Weibull random variable u (wind speed) is scaled or shaped therefore, there is no need of summing the power constant [5, 10].

$$L(p,b) = \prod_{i=1}^{n} f(u_i) = \left(\frac{b^n}{p^n}\right)\left(\prod_{i=1}^{n} \frac{u_i}{p}\right)^{b-1} \exp\left(-\sum_{i=1}^{n} \frac{u_i}{p}\right)^{b} \quad (12)$$

By taking the natural logarithm transformation, we have the equation

$$\ln L(p,b) = n\ln b - n\ln p + (b-1)\frac{\sum_{i=1}^{n}\ln(u_i)}{p} + \left(-\sum_{i=1}^{n} \frac{u_i}{p}\right)^{b} \quad (13)$$

$$\ln L(p,b) = n\ln b - n\ln p + \frac{(b-1)}{p}\sum_{i=1}^{n}\ln(u_i) + \left(-\frac{1}{p}\sum_{i=1}^{n} u_i^{b}\right)$$

Differentiating ln L (p, b) with respect to p, we obtain

$$\frac{\partial}{\partial p}\ln L(p,b) = -\frac{n}{p} - \frac{1}{p^2}(b-1)\sum_{i=1}^{n}\ln(u_i) + \frac{1}{p^2}\sum_{i=1}^{n} u_i^{b} \quad (14)$$

Differentiating ln L (p, b) with respect to b, we obtain

$$\frac{\partial}{\partial b}\ln L(p,b) = \frac{n}{b} + \frac{\sum_{i=1}^{n}\ln(u_i)}{p} - \frac{1}{p}\sum_{i=1}^{n} u_i^{b}\ln(u_i) \quad (15)$$

Equating equations (14) and (15) to zero gives the maximum likelihood estimates $(\hat{p}, \hat{b})$ of (p, b). The estimate for $\hat{p}$ is as shown

$$-\frac{n}{p} - \frac{1}{p^2}(b-1)\sum_{i=1}^{n}\ln(u_i) + \frac{1}{p^2}\sum_{i=1}^{n}u_i^b = 0 \quad (16)$$

$$\frac{n}{p} = \frac{1}{p^2}(b-1)\sum_{i=1}^{n}\ln(u_i) + \frac{1}{p^2}\sum_{i=1}^{n}u_i^b \quad (17)$$

$$\frac{n}{p} = -\frac{1}{p^2}\left[(b-1)\sum_{i=1}^{n}\ln(u_i) - \sum_{i=1}^{n}u_i^b\right] \quad (18)$$

$$\frac{n}{p}p^2 = -\left[(b-1)\sum_{i=1}^{n}\ln(u_i) - \sum_{i=1}^{n}u_i^b\right] \quad (19)$$

$$\hat{p} = \left.-\left[(b-1)\sum_{i=1}^{n}\ln(u_i) - \sum_{i=1}^{n}u_i^b\right]\right/n \quad (20)$$

The estimate for $\hat{b}$ is obtained as;

$$\frac{n}{b} + \frac{\sum_{i=1}^{n}\ln(u_i)}{p} - \frac{1}{p}\sum_{i=1}^{n}u_i^b\ln(u_i) = 0 \quad (21)$$

$$\frac{n}{b} + \frac{\sum_{i=1}^{n}\ln(u_i)}{p} = \frac{1}{p}\sum_{i=1}^{n}u_i^b\ln(u_i) \quad (22)$$

Further solution to find $\hat{b}$ is given as;

$$\frac{n}{\hat{b}} = \frac{1}{p}\sum_{i=1}^{n}u_i^{\hat{b}}\ln(u_i) - \frac{\sum_{i=1}^{n}\ln(u_i)}{p}$$

$$\frac{n}{\hat{b}} = \frac{1}{p}\left[\sum_{i=1}^{n}u_i^{\hat{b}}\ln(u_i) - \sum_{i=1}^{n}\ln(u_i)\right]$$

Substituting $\hat{p}$ from equation (22), gives;

$$\frac{n}{\hat{b}} = \frac{\sum_{i=1}^{n}u_i^{\hat{b}}\ln(u_i) - \sum_{i=1}^{n}\ln(u_i)}{-\left[\left(\left(\hat{b}-1\right)\sum_{i=1}^{n}\ln(u_i)\right) - \left(\sum_{i=1}^{n}u_i^{\hat{b}}\right)\right]/n} \quad (23)$$

Introduce logarithm to eliminate the power $\hat{b}$ in equation (23)

$$\frac{n}{\hat{b}} = \frac{\sum_{i=1}^{n}\ln u_i^{\hat{b}}\ln(u_i) - \sum_{i=1}^{n}\ln(u_i)}{-\left[\left(\left(\hat{b}-1\right)\sum_{i=1}^{n}\ln(u_i)\right) - \left(\sum_{i=1}^{n}\ln u_i^{\hat{b}}\right)\right]/n} \quad (24)$$

$$\frac{n}{\hat{b}} = \frac{\sum_{i=1}^{n}\hat{b}\ln(u_i)\ln(u_i) - \sum_{i=1}^{n}\ln(u_i)}{-\left[\left(\left(\hat{b}-1\right)\sum_{i=1}^{n}\ln(u_i)\right) - \left(\sum_{i=1}^{n}\hat{b}\ln(u_i)\right)\right]/n} \quad (25)$$

Factorizing equation (25) gives;

$$\frac{n}{\hat{b}} = \frac{\hat{b}\ln(u_i)\left[\sum_{i=1}^{n}\ln(u_i)-1\right]}{-\left(\hat{b}-1\right) + \hat{b}\left[\sum_{i=1}^{n}\ln(u_i)\right]/n} \quad (26)$$

$$\frac{n}{b^2\ln(u_i)} = \frac{\left[\sum_{i=1}^{n}\ln(u_i)-1\right]}{\left[\sum_{i=1}^{n}\ln(u_i)\right]/n} \quad (27)$$

$$\frac{n}{b^2} = \left(\frac{\left(\sum_{i=1}^{n}\ln(u_i)-1\right)}{\left(\sum_{i=1}^{n}\ln(u_i)\right)/n}\right)\bullet\left(\ln(u_i)\right) \quad (28)$$

$$\frac{1}{b^2} = \frac{\left[\left(\frac{\left(\sum_{i=1}^{n}\ln(u_i)-1\right)}{\left(\sum_{i=1}^{n}\ln(u_i)\right)/n}\right)\bullet\left(\ln(u_i)\right)\right]}{n} \quad (29)$$

$$\hat{b}^2 = \frac{n}{\left[\left(\frac{\left(\sum_{i=1}^{n}\ln(u_i)-1\right)}{\left(\sum_{i=1}^{n}\ln(u_i)\right)/n}\right)\bullet\left(\ln(u_i)\right)\right]} \quad (30)$$

$$\hat{b} = \sqrt{\frac{n}{\left[\left(\frac{\left(\sum_{i=1}^{n}\ln(u_i)-1\right)}{\left(\sum_{i=1}^{n}\ln(u_i)\right)/n}\right)\bullet\left(\ln(u_i)\right)\right]}} \quad (31)$$

ii. MLE for Lognormal Distribution with 2-P

The density function for the two-parameter log-normal distribution with two parameters v and k given as [3, 4]:

$$f(p) = \frac{1}{k\sqrt{2p\pi}} \exp \frac{(\ln p - v)^2}{2k^2} \tag{32}$$

To compute the maximum likelihood, we obtain the likelihood function first. The likelihood function of lognormal distribution for series of $p_{is}(i = 1, 2, \ldots, n)$ is derived by taking the product of probability density of the individual p_{is} given as below.

$$L(v, k^2) = \prod_{i=1}^{n} f(p) = \prod_{i=1}^{n} \left(\left(2\pi k^2\right)^{-\frac{1}{2}} p_i^{-1} \exp\left[\frac{-\left(\ln(p_i) - v\right)^2}{2k^2} \right] \right) = \left(2\pi k^2\right)^{-\frac{n}{2}} \prod_{i=1}^{n} p_i^{-1} \exp\left[\sum_{i=1}^{n} \frac{-\left(\ln(p_i) - v\right)^2}{2k^2} \right] \tag{33}$$

We then derive the likelihood function by taking the natural logarithm

$$\ln L\left(v, k^2\right) = \ln\left(\left(2\pi k^2\right)^{-\frac{n}{2}} \prod_{i=1}^{n} p_i^{-1} \exp\left[\sum_{i=1}^{n} \frac{-\left(\ln(p_i) - v\right)^2}{2k^2} \right] \right)$$

$$= -\frac{n}{2}\ln\left(2\pi k^2\right) - \sum_{i=1}^{n}\ln\left(p_i\right) - \frac{\sum_{i=1}^{n}\left(\ln(p_i) - v\right)^2}{2k^2}$$

$$= -\frac{n}{2}\ln\left(2\pi k^2\right) - \sum_{i=1}^{n}\ln\left(p_i\right) - \frac{\sum_{i=1}^{n}\left[\ln(p_i)^2 - 2\ln(p_i)v + v^2\right]}{2k^2}$$

$$= -\frac{n}{2}\ln\left(2\pi k^2\right) - \sum_{i=1}^{n}\ln\left(p_i\right) - \frac{\sum_{i=1}^{n}\ln(p_i)^2}{2k^2} + \frac{\sum_{i=1}^{n}2\ln(p_i)v}{2k^2} - \frac{\sum_{i=1}^{n}v^2}{2k^2}$$

$$= -\frac{n}{2}\ln\left(2\pi k^2\right) - \sum_{i=1}^{n}\ln\left(p_i\right) - \frac{\sum_{i=1}^{n}\ln(p_i)^2}{2k^2} + \frac{\sum_{i=1}^{n}\ln(p_i)v}{k^2} - \frac{nv^2}{2k^2} \tag{34}$$

To find $\hat{v}$ and $\hat{k}^2$, maximize $lnL(v, k^2)$. To find this, we differentiate equation (34) with respect to v and k^2 by setting the equation equal to 0: with respect to v, to obtain

$$\frac{\partial}{\partial v}\ln L\left(v, k^2\right) = \frac{\sum_{i=1}^{n}\ln(p_i)}{k^2} - \frac{2nv}{2k^2} = 0 \Rightarrow \frac{\sum_{i=1}^{n}\ln(p_i)}{k^2} = \frac{nv}{k^2} \Rightarrow nv = \sum_{i=1}^{n}\ln(p_i) \Rightarrow \hat{v} = \frac{\sum_{i=1}^{n}\ln(p_i)}{n} \tag{35}$$

With respect to $\hat{k}^2$, we obtain,

$$\frac{\partial}{\partial k^2}\ln L\left(v,k^2\right)=-\frac{n}{2}\frac{1}{k^2}-\frac{\sum_{i=1}^{n}\left(\ln\left(p_i\right)-v\right)^2}{2}\left(-k^2\right)^2=-\frac{n}{2k^2}+\frac{\sum_{i=1}^{n}\left(\ln\left(p_i\right)-v\right)^2}{2\left(k^2\right)^2}=0$$

$$\Rightarrow \frac{n}{2k^2}=\frac{\sum_{i=1}^{n}\left(\ln\left(p_i\right)-v\right)^2}{2k^4}\Rightarrow n=\frac{\sum_{i=1}^{n}\left(\ln\left(p_i\right)-v\right)^2}{k^2}$$

$$\Rightarrow k^2=\frac{\sum_{i=1}^{n}\left(\ln\left(p_i\right)-v\right)^2}{n}\Rightarrow k^2=\frac{\sum_{i=1}^{n}\left(\ln\left(p_i\right)-\frac{\sum_{i=1}^{n}\ln\left(p_i\right)}{n}\right)^2}{n} \qquad (36)$$

iii. MLE for Gamma Distribution with 2-P

In this section, we considered also a gamma distribution with shape parameter and scale parameter since it is the distribution which is widely used in real life data sets. The probability density function of gamma random variable y in combination with two parameters z and q representing the shape and scale parameters respectively is given by [4, 9, 12, 24].

$$f\left(y,z,q\right)=\frac{y^{z-1}}{\Gamma(z)q^z}\exp\left(-\frac{y}{q}\right),\left(y,z,q>0\right) \qquad (37)$$

Where:

$$\Gamma(v)=\int_0^{\infty}y^{v-1}\exp^{-y}dy,\left(v>0\right) \qquad (38)$$

For maximum likelihood estimation, we first get the likelihood function which is given by [18]:

$$L\left(z,q\right)=\prod_{i=1}^{n}f\left(y,z,q\right)=\prod_{i=1}^{n}\frac{y_i^{z-1}\exp\left(-\frac{y_i}{q}\right)}{\Gamma(z)q^z} \qquad (39)$$

The log of the likelihood function is given by

$$\ln L\left(z,q\right)=\sum_{i=1}^{n}\ln\left[\frac{y_i^{z-1}\exp\left(-\frac{y_i}{q}\right)}{\Gamma(z)q^z}\right]$$

$$=\sum_{i=1}^{n}\left[\ln\left(\frac{1}{\Gamma(z)q^z}\right)+\ln\left(y^{z-1}\exp\left(-\frac{y}{q}\right)\right)\right]$$

$$=\sum_{i=1}^{n}\ln\left(\frac{1}{\Gamma(z)q^z}\right)+\sum_{i=1}^{n}\ln\left(y^{z-1}\exp\left(-\frac{y}{q}\right)\right)$$

Since, $ln(q^z) = zln(q)$, we obtain

$$=-\sum_{i=1}^{n}\left[\ln\left(\Gamma(z)\right)+z\ln(q)\right]+\sum_{i=1}^{n}\left[(z-1)\ln\left(y_i\right)-\frac{y_i}{q}\right]$$

Therefore, we have;

$$=-n\left[\ln\left(\Gamma(z)\right)+z\ln(q)\right]+(z-1)\sum_{i=1}^{n}\ln\left(y_i\right)-\frac{1}{q}\sum_{i=1}^{n}y_i \qquad (40)$$

To find the maximum likelihood estimates for $\hat{z}$ and $\hat{q}$ for z and q, we equate equation (40) to zero and then find out the partial derivatives with respect to $\hat{z}$ and $\hat{q}$ respectively.

$$\frac{\partial}{\partial z}\ln L\left(z,q\right)=-n\left[\ln(q)+\frac{\Gamma'(z)}{\Gamma(z)}\right]+\sum_{i=1}^{n}\ln\left(y_i\right)=0$$

$$\Rightarrow \hat{z}=\ln(q)+\frac{\Gamma'(z)}{\Gamma(z)}=\frac{\sum_{i=1}^{n}\ln\left(y_i\right)}{n} \qquad (41)$$

Differentiating with respect to $\hat{q}$ and setting the equation equal to 0: to get;

$$\frac{\partial}{\partial q} \ln L(z,q) = -n\frac{z}{q} + \frac{1}{q^z}\sum_{i=1}^{n} y_i = 0$$

$$= \frac{nz}{q} = \frac{1}{q^z}\sum_{i=1}^{n} y_i \Rightarrow \frac{z}{q} = \frac{1}{q^z}\left(\sum_{i=1}^{n} y_i \middle/ n\right) \Rightarrow \frac{z}{q} = \frac{1}{q^z}\bar{y}$$

$$\Rightarrow \hat{q} = \frac{\bar{y}}{\hat{z}} \tag{42}$$

Therefore, we have;

$$\hat{q} = \frac{\sum_{i=1}^{n} y_i \middle/ n}{\sum_{i=1}^{n} \ln(y_i) \middle/ n} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} \ln(y_i)} \tag{43}$$

iv. MLE for 3-P Probability Distributions

The three parameters are shape, scale and threshold parameter. The three parameter distributions applied on this study are Weibull, gamma and log-normal and are formulated as given in equations (4), (5) and (6) respectively.

The third parameter called threshold parameter is also known as the location parameter which determines where to shift the 3-p density function along the X-axis. The threshold parameter locates the distribution along the time scale and has same units as the distribution variable units. This third parameter is used to try to fit the data point into a straight line when the initial data do not fall on a straight line. It was therefore used to transform the data set to fit or resemble the hypothesized distribution better. After obtaining the threshold parameter, it is subtracted from the original data

and obtain a new data set which is then used to estimate the other two parameters (shape and scale parameters). Since the threshold parameter value is not constant, the AIC and BIC are used to estimate the threshold parameter. The threshold value with the lowest AIC and BIC values was considered to be the efficient and precise for further analysis. It was subtracted from the original data set and the resulting data set was then used for estimating the scale and shape parameters for both Weibull, Log-normal and Gamma probability distributions with 3-p using the same maximum likelihood estimates obtained for the 2-p under each of the three distributions.

2.3.2. Minimum Distance Estimation (MDE)

According to [19], the minimum distance method reduces the computational complexity since it omits the Jacobian element which is usually present in the likelihood function.

The method of minimum distance estimation depends on the test statistics of Anderson-Darling (AD) test [12]. The expression for Anderson-Darling based on minimum distance estimation is formulated as follows;

$$AD = A_n^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left(\log\left(y_{i:n}\right) + \log\left(1 - y_{(n+1-i)}\right)\right) \tag{44}$$

For the 2-p estimation for Weibull, gamma and log-normal using Minimum Distance Estimation method, the study uses the Anderson-Darling Minimum Distance estimator.

Anderson-Darling Estimation Method: The MDE technique is based on the application of Anderson-darling statistics and is defined as Anderson-Darling estimator (ADE). A study developed this test as an alternative to statistical test to be used significantly to examine sample distribution departure from normality [1]. By applying the Anderson-Darling test statistics, we can obtain the Anderson-Darling estimates $\hat{M}_{ADE}$ and $\hat{K}_{ADE}$ representing the scale and shape parameter estimates respectively for the three distributions from the following equation

$$A(m,k) = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left(\log F\left(y_{i:n} \backslash m,k\right) + \log V\left(y_{(n+1-i)} \backslash m,k\right)\right) \tag{45}$$

The estimates $\hat{M}_{ADE}$ and $\hat{K}_{ADE}$ are obtained by minimizing equation (45) with respect to m and k. Similarly, these estimates can be obtained from the solution of the following non-linear equations [11, 22].

$$\sum_{i=1}^{n}(2i-1)\left[\frac{\Delta_1\left(y_{i:n} \backslash m,k\right)}{F\left(y_{i:n} \backslash m,k\right)} - \frac{\Delta_1\left(y_{(n+1-i)} \backslash m,k\right)}{V\left(y_{(n+1-i)} \backslash m,k\right)}\right] = 0 \tag{46}$$

$$\sum_{i=1}^{n}(2i-1)\left[\frac{\Delta_2\left(y_{i:n} \backslash m,k\right)}{F\left(y_{i:n} \backslash m,k\right)} - \frac{\Delta_2\left(y_{(n+1-i)} \backslash m,k\right)}{V\left(y_{(n+1-i)} \backslash m,k\right)}\right] = 0 \tag{47}$$

Where $\Delta_1(|m,k)$ and $\Delta_2(|m,k)$ in equation (46) and (47) are given as;

$$\Delta_1\left(y_{i:n} \backslash m,k\right) = \frac{\partial}{\partial m} F\left(y_{i:n} \backslash m,k\right) = \frac{\exp^{ky_i - 1}}{\left(\exp^{ky_i} - 1 + m\right)^2}$$

$$\Delta_2\left(y_{i:n} \backslash m,k\right) = \frac{\partial}{\partial k} F\left(y_{i:n} \backslash m,k\right) = \frac{ky\exp^{ky_i - 1}}{\left(\exp^{ky_i} - 1 + m\right)^2}$$

For the 3-p estimation, after getting the threshold value we apply the same Anderson-Darling estimation technique to get

the other two parameters namely; shape and scale parameters.

2.4. Comparison of Efficiency Between MLE And MDE Techniques

2.4.1. Comparison of the Probability Distributions

In determining the best distribution for the wind speed among the fitted distribution so that it could be used in assessing the efficiency of the estimation technique, the study Used the Akaike's Information Criteria and Bayesian Information Criterion. The techniques were applied as follows;

i. Akaike's Information Criterion (AIC)

The Akaike's Information Criterion is calculated as shown below

$$AIC = -2logL(p) + 2w \qquad (48)$$

Where log L(P) defines the value of the maximized log-likelihood objective function for a model with w parameters. A smaller AIC value represents a better fit [25].

ii. Bayesian Information Criterion (BIC)

The Bayesian Information Criterion is calculated as below

$$BIC = -2logL(p) + wlogM \qquad (49)$$

Where log L(P) represents the values of the maximized log-likelihood objective function for a model with w parameters fit to M data points [25]. A smaller Bayesian Information Criterion value indicates a better fit (best model for fitting the data).

2.4.2. Comparison of Estimation Technique

An estimator is said to be more efficient than another estimator if it is more reliable and precise for the same sample size. For the research to achieve part of its specific objectives, there is need to understand how efficiency test is carried out. This was achieved using relative efficiency test. According to Dookie, I. and Gupta, R. [6, 9] study it is said that the method of MLE is popularly applied because its estimators are generally asymptotically consistent and unbiased. From the study conducted by Galvao, F. and Maleki, F. [7, 13], it is concluded that MDE method is also unbiased estimation method. Since the two studies concluded that both MLE and MDE are unbiased, the relative efficiency test is given as follows;

$$Relative\ Efficiency\ (R.E) = \frac{MSE(MLE)}{MSE(MDE)} = \frac{MSE(\hat{X}_1)}{MSE(\hat{X}_2)} = \frac{Var(\hat{X}_1)}{Var(\hat{X}_2)} \qquad (50)$$

Where $\hat{X}_1$ and $\hat{X}_2$ are the estimators under MLE and MDE respectively.

If the ratio is less than 1, implies that MLE is more efficient (have smaller mean square error) and the estimator are therefore unbiased, sufficient and consistent. If the is greater than 1, then it indicates that MDE is more efficient meaning that it has small mean square error and therefore its estimates are unbiased, consistent and sufficient.

The relative efficiency for Weibull distribution, gamma distribution and log-normal distribution are as given in Table 1.

***Table 1.** Relative Efficiency formulas.*

Distribution	Relative Efficiency
Gamma	$\frac{z_1 q_2^2}{z_2 q_1^2}$
Lognormal	$\frac{[\exp k_1^2 - 1]\exp(2v_1 - k_1^2)}{[\exp k_2^2 - 1]\exp(2v_2 - k_2^2)}$
Weibull	$\frac{p_2^{2/b_2}}{p_1^{2/b_1}}$

For the three parameter distribution, the relative efficiency formulations were the same as the formulations for two parameter distributions because the threshold value is the constant for both MLE and MDE techniques and if the threshold is introduced in the ratio, it will cancel itself making the relative efficiency formula for 3-p to fall back to the relative efficiency formula for 2-p.

3. Results

3.1. Data Description

From the analysis of 66858 observations, the mean wind speed was determined to be 2.1617 m/s with the standard deviation of 1.5124m/s. However, given that the region experience slight weather conditions the data recorded several outlier observations and as a results the study removed the outlier observations and after removing the outliers, the statistics were as illustrated in table 2.

***Table 2.** Summary Statistics after Removing the Outliers.*

Min value	0.12
Max value	5.35
Estimated Mean	1.965777
Estimated Median	1.62
Estimated std	1.24065
Estimated kurtosis	2.809401
Estimated skewness	0.8433485

Based on the results in table 2, the minimum speed recorded was 0.12 m/s and the maximum speed was 5.35 m/s. The average wind speed was 1.9658 m/s with a standard deviation of 1.2407 m/s.

3.2. Fitting of Probability Distribution Using MLE Technique

3.2.1. Two Parameter Distributions

After removal of the outlier observations in the data set, the study fitted a two parameter probability distributions for the wind speed data using the Maximum Likelihood Estimation technique, the results of the fitted distribution parameter were as illustrated in table 3;

The results in table 2, shows that all the parameters were significantly different from zero, this shows that the fitted distributions were significantly adequate. The comparison of the fitted distributions against the underlying data set was as illustrated by Figure 1;

***Table 3.** Two parameter distributions using MLE technique.*

Distribution	Parameter	Estimate	Std Error	t stat	95% t critical value
Weibull	Shape	1.670	0.005	326.853	1.96
	Scale	2.211	0.006	398.789	
Gamma	Shape	2.476	0.013	189.889	1.96
	Scale/rate	1.260	0.007	171.323	
Log-normal	Shape	0.461	0.003	168.730	1.96
	Scale	0.690	0.002	357.080	
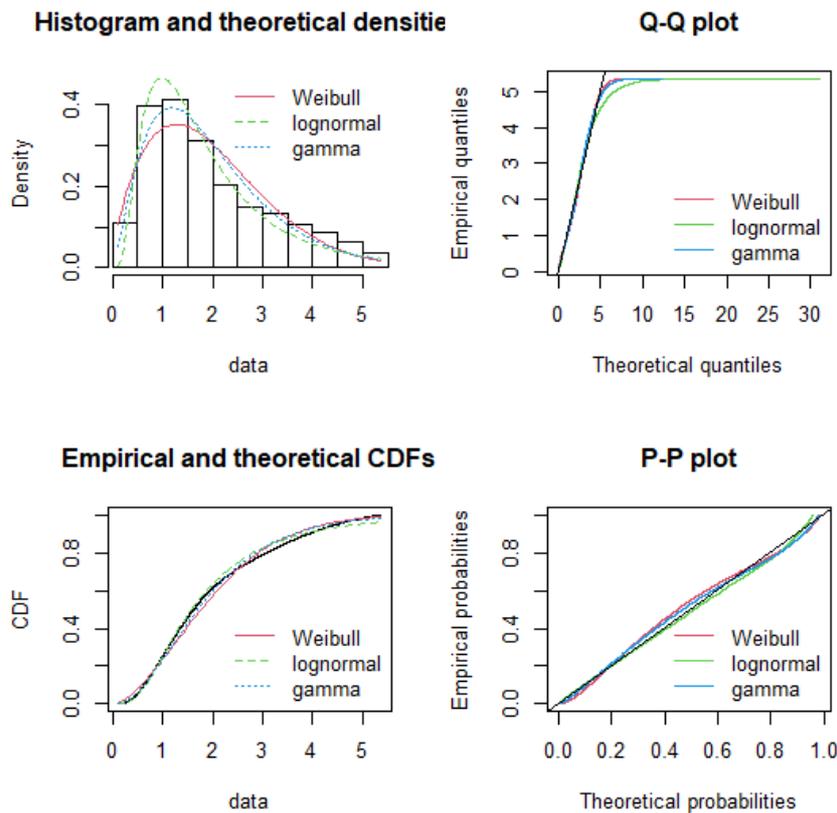
***Figure 1.** Wind distribution in comparison to fitted two parameter distributions.*

The results in Figure 1, shows that the three fitted probability distributions in one way or the other fit the distribution of the underlying wind speeds.

3.2.2. Three Parameter Distributions

The study also fitted three parameter probability distribution to the wind speed using MLE technique and different threshold values, the results of comparison of different threshold values were as illustrated in table 4;

***Table 4.** Comparison of different threshold value.*

Threshold value	Distribution	AIC	BIC
0.1170	Weibull	190007.3	190025.4
	Gamma	189799.8	189817.9
	Log-normal	195747.6	195765.8
0.1174	Weibull	190003.8	190021.9
	Gamma	189803	189821.1
	Log-normal	195803	195821.1
0.1175	Weibull	190002.9	190021
	Gamma	189803.9	189822
	Log-normal	195817.8	195835.9

Threshold value	Distribution	AIC	BIC
0.1180	Weibull	189999.1	190017.2
	Gamma	189809.1	189827.2
	Log-normal	195898.9	195917.1
0.1185	Weibull	189996.1	190014.3
	Gamma	189816	189834.1
	Log-normal	195997.8	196015.9
0.1190	Weibull	189994.7	190012.9
	Gamma	189826	189844.1
	Log-normal	196129.8	196147.9
0.1195	Weibull	189997.1	190015.2
	Gamma	189843.4	189816.5
	Log-normal	196347	196365.2
0.1199	Weibull	190012.5	190030.7
	Gamma	189884.1	189902.2
	Log-normal	196869	196887.1

From Table 4 it can be observed that as the threshold value increases, the AIC and BIC for gamma and log-normal distributions increases while for Weibull distribution the AIC and BIC value is showing slight change in the trend since they are almost rotating at nearly the same value. From the

table it can be seen that in almost all aspects gamma distribution is reporting lower AIC and BIC values with the lowest AIC and BIC value observed under the threshold value of 0.1174 with AIC value of 189803 and BIC value of 189821.1.

Since speed cannot be negative as per the threshold for log-normal and also that gamma appeared to be having smaller AIC and BIC values for all the tested threshold values, the study opted to use the threshold value for gamma (0.1174 m/s) as the threshold value for the rest of the analysis under maximum likelihood method.

After transformation of the data using the threshold value (subtracting the threshold value from each observation), the study fitted the other two parameter distributions for the respective distributions using the MLE technique for two parameter distributions. The results of the fitted distribution were as illustrated in the table 5;

***Table 5.** Parameters for 3-P probability distributions.*

Distribution	Parameter	Estimate
Weibull	Shape	1.5394
	Scale	2.0592
	Threshold	0.1174
Gamma	Shape	2.0718
	Scale	1.1209
	Threshold	0.1174
Log-normal	Shape	0.3600
	Scale	0.7883
	Threshold	0.1174

3.3. Fitting Wind Speed Data to Probability Distribution Using MDE

3.3.1. Two Parameter Probability Distributions

The wind data set was fitted to three probability distribution (Weibull, Log-normal and Gamma). The graphical illustration of the fitted probability distributions was as illustrated in Figure 2.
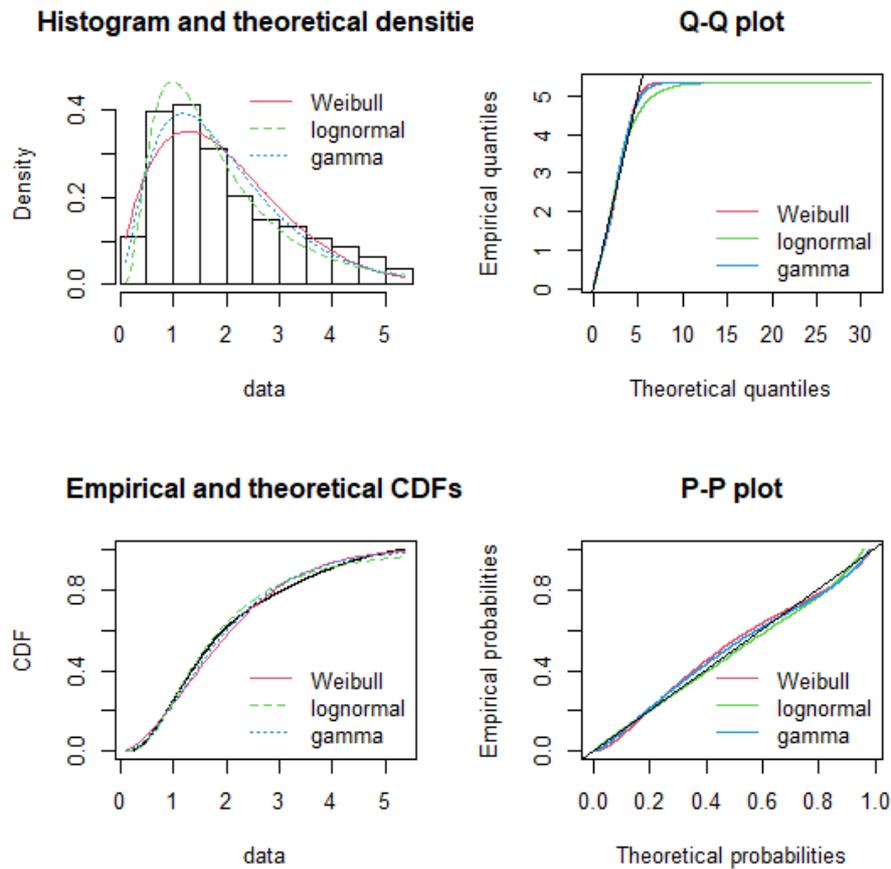

***Figure 2.** Graphical outputs for wind speed data.*

From Figure 2, under histogram, it can be seen that the log-normal distribution is more peaked than the gamma and the Weibull distribution even though all of them are positively skewed. From the frequency polygons, it can be seen that the distribution of log-normal have less cases of under estimation and over estimation of the data hence log-normal is considered in this case the best distribution.

By considering the Q-Q plots in Figure 2, it can be observed that all the three distributions are almost estimating the same values from the minimum speed to a speed of 4 m/s. Above speed of 4 m/s, deviations can be observed but Weibull distribution seems to deviate less from the best line of fit. Therefore, if we are to make our verdict using Q-Q plots then Weibull distribution can be marked as the best distribution for studying the data.

From the Cumulative density function graph, it can be observed that the speeds above 4 m/s cannot be probabilistically predicted accurately due to the deviations

experienced but it can be seen from the curves that log-normal is much closer to the curve representing the observed data than Weibull and gamma distributions (for speeds below 4 m/s).

For the P-P plots, the deviations are observed but log-normal looks more close to the best line of fit with less deviations compared to Weibull and gamma distributions.

From the graphical analysis it can be seen that most of the plots/graphs like histogram, PP plots and CDF graph displays log-normal as the best distribution for fitting the data.

Test of Goodness of Fit Analysis: The estimated parameters for the distributions was using minimum distance technique was as shown in table 6.

Table 6. *Estimated parameters for 2-P distributions using MDE.*

Distribution	Parameter	Estimate
Weibull	Shape	1.502943
	Scale	2.172747
Gamma	Shape	2.107526
	Scale	1.062046
Log-normal	Shape	0.490416
	Scale	0.68618

The comparison of the distributions in terms of Goodness of fit was carried out and the results were as illustrated in table 7;

Table 7. *Test of goodness of fit using MDE for Two Parameter Distributions.*

Statistics	Weibull	Gamma	Log-normal
Kolmogorov-Smirnov Criteria	0.051438	0.031019	0.041869
AIC	192915.1	191315.6	192463.5
BIC	192933.3	191333.7	192481.6

Using the Kolmogorov-Smirnov statistic it can be confirmed that the data followed all the three distributions namely Weibull, gamma and log-normal since all of them give statistics test values lower than the critical value (0.136), this lead to a decision of not rejecting the null hypothesis for all the three distributions. Gamma fits the data best because from the AIC (191315.6) and BIC (191333.7) it is clearly evidenced that the distribution with smaller values is gamma distribution and therefore as per the decision rule it is considered the best of the three distributions for fitting the wind speed data.

Goodness of fit test is considered to be more accurate method for identifying the best distribution compared to the graphical methods because graphical methods are not most precise figures when it comes to estimating the value of each distribution as seen with goodness of fit test. Therefore, it is concluded that by using the distance technique the gamma distribution is the best distribution for fitting the wind speed data and examining its characteristics with the AIC value of 191315.6 and BIC value of 191333.7.

3.3.2. Three Parameter Probability Distributions Analysis
i. Determination of Appropriate Threshold Value
Before fitting the three parameter distributions using MDE, different threshold values were tested to determine the most appropriate, the results of comparison of the different threshold values were as illustrated in table 8;

Table 8. *Determination of Threshold value for MDE.*

Threshold value	Distribution	AIC	BIC
0.1170	Weibull	190791	190809.1
	Gamma	190226.4	190244.6
	Log-normal	196818.8	196837
0.1174	Weibull	190785.2	190803.3
	Gamma	190227.2	190245.3
	Log-normal	196888.9	196907
0.1180	Weibull	190777.2	190795.3
	Gamma	190228.9	190247
	Log-normal	197010.2	197028.3
0.1185	Weibull	190771.5	190789.6
	Gamma	190232	190250.1
	Log-normal	197133.1	197151.3
0.1190	Weibull	190766.7	190784.9
	Gamma	190237.3	190255.4
	Log-normal	197296.9	197315
0.1195	Weibull	190764.9	190783
	Gamma	190248.9	190267.1
	Log-normal	197561.8	197580
0.1199	Weibull	190773.6	190791.8
	Gamma	190279.	190298
	Log-normal	198192.7	198210.8

The results in table 8 showed that gamma distribution had the smallest AIC and BIC value under all tested threshold values. Therefore, comparing the AIC and BIC values of gamma distribution, it was determined that the gamma distribution with the threshold value of 0.1174 had the lowest AIC and BIC, therefore the threshold of 0.1174 was the best.

ii. Graphical Analysis
It is also important to understand the distribution of the wind data under these three distributions of interest hence it is very important to also study some graphical distributions.

From Figure 3, on the histogram, we can see that even though log-normal is more peaked than gamma and Weibull, it shows larger deviations compared to gamma and Weibull. Since gamma is more peaked than Weibull and its deviations are not much compared to log-normal, it can be picked as the best distribution for fitting this data.

For the P-P plot on Figure 3, gamma shows slight deviation which is almost uniform from the line of best fit hence from the PP lot distribution gamma is the best.

From Figure 3 on the QQ plot, all the three distributions are fitting the data well. The deviations from the line of best fit is observed to start at speed above 3.5 m/s with Weibull showing less deviation. Therefore, using QQ plot Weibull is observed to be the best distribution for the study.

For the CDFs graph, gamma shows less deviations from the best line for almost 80 percent of the wind speed data compared to log-normal and Weibull distributions hence it is termed the best distribution since it can be used to probabilistically examine 80 percent of the data.

From the graphical analysis it can be concluded that gamma distribution is the best for studying this regions data since from the four graphs displayed three of them exposed gamma as the best distribution (histogram, PP plot and CDFs graph).
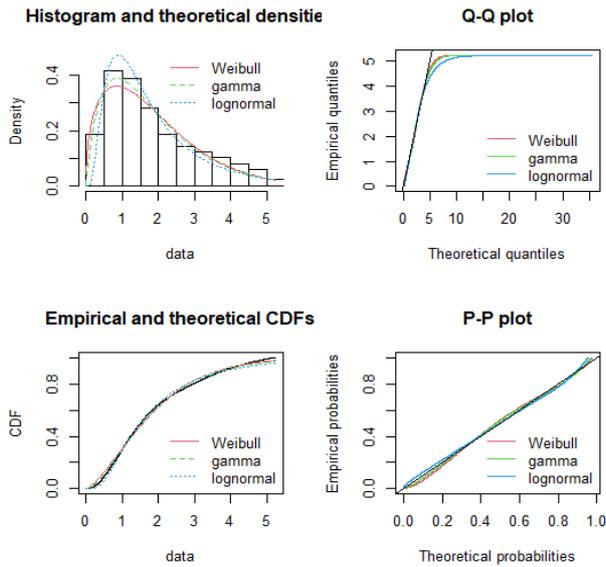
Figure 3. *Graphical output after subtracting the threshold value.*

iii. Statistical Analysis

In this section we looked at both statistical analysis and the graphical representation of each distribution. First, there is need to investigate how the AIC and BIC behave under different value for the threshold parameter. Using the threshold value of 0.1174, the other two parameters for all the three distributions are given in Table 9.

Table 9. *Scale and shape parameters for 3-P distributions.*

Distribution	Parameter	Estimate
Weibull	Shape	1.4115
	Scale	2.038586
Gamma	Shape	1.864567
	Scale	0.993702
Log-normal	Shape	0.4091521
	Scale	0.732395

To be sure that our data followed this three specific distributions, we performed a statistical tests using the goodness of fit statistics. The goodness of fit statistics applied are summarized as shown in the Table 10.

Table 10. *K-S statistics for 3-P distributions.*

Statistics	Weibull	Gamma	Log-normal
Kolmogorov-Smirnov	0.038007	0.028086	0.044478

From Table 10; it can be confirmed that all the three distributions that were recommended under Cullen and Frey graph, are still found to be statistically significant from the analysis using Kolmogorov-Smirnov test. This is clearly supported by the fact that all the Kolmogorov-Smirnov statistics values are less than the critical value 0.136, accepting the null hypothesis.

Therefore, using statistical analysis it can be summarized that gamma three parameter distribution is the best among the three distributions for studying the wind speed data.

Using the minimum distance estimation method, we therefore conclude that gamma three parameter distribution is the best with the following characteristics in table 11.

Table 11. *Best distribution using MDE.*

Distribution	Criteria	Estimate
Gamma	AIC	190227.2
	BIC	190245.3
	Parameters	
	Threshold	0.1174
	Shape	1.864567
	Scale	0.993702

3.4. Comparison of the Probability Distributions

The precision of this methods is based on the decision rule that the best method is the one which gives smaller AIC and BIC values.

By comparing the maximum likelihood method and the minimum distance method, we choose the method with the smaller AIC and BIC value for their best distributions which will be termed as the best distribution for studying the wind speed data.

From the Table 6, it can be observed that maximum likelihood estimation method yields smaller values for both AIC and BIC for both the 2-parameter distribution and the three-parameter distribution. This leads us to a conclusion that the maximum likelihood estimation is the best estimation technique and the best distribution is gamma in both 2-parameter and 3-parameter distributions.

Lastly to know the best distribution between the two parameter and the three parameter we again compare their AIC and BIC value under maximum likelihood method since it is the best estimation technique. The decision rule relies on the distribution with the smaller AIC and BIC value. Since in both cases of two and three parameter distribution analysis we have gamma as the best distribution, we now compare the AIC and BIC for this 2-parameter gamma distributions. The comparison is as given in Table 12.

Table 12. *Model comparison.*

Method	Parameters	Distribution	Criteria	Value
Maximum likelihood method	Two-parameter	Weibull	AIC	191777.5
			BIC	191795.7
		Log-normal	AIC	192340.2
			BIC	192358.4
		Gamma	AIC	190407.2
			BIC	190425.3
Minimum Distance method	Two-parameter	Weibull	AIC	192915.1
			BIC	192933.3
		Log-normal	AIC	192463.5
			BIC	192481.6
		Gamma	AIC	191315
			BIC	191333.7
Maximum likelihood method	Three parameters	Weibull	AIC	190003.8
			BIC	190021.9
		Log-normal	AIC	195803
			BIC	195821.1
		Gamma	AIC	189803
			BIC	189821.1
Minimum Distance method	Three parameters	Weibull	AIC	190785.2
			BIC	190803.3
		Log-normal	AIC	196888.9
			BIC	196907
		Gamma	AIC	190227.2
			BIC	190245.3

Table 13. Best distributions for 2-P and 3-P.

Distribution	Criteria	Value
Gamma two parameter	AIC	190407.2
	BIC	190425.3
Gamma three parameter	AIC	189803
	BIC	189821.1

From Table 13, gamma distribution with 3 parameters has smaller AIC and BIC value compared to gamma distribution with 2-p. Therefore, gamma distribution with 3-p is the best distribution for examining wind speed data. This distribution has the following defined parameters given in Table 14.

Table 14. Best distribution estimates.

Distribution	Parameter	Estimate
Gamma	Threshold	0.1174
	Shape	2.071773
	Scale	1.120855

$$f(y,z,q,t) = \frac{y\,0.1174^{2.071773-1}}{\Gamma(2.071773/0.1174)1.120855^{2.071773}}\exp\left[-\left(\frac{y}{1.120855}\right)^{0.1174}\right] \quad (52)$$

Where;

y>0 is the hourly wind speed data,

And Γ is a continuous gamma function given a;

$$\Gamma(v) = \int_0^\infty y^{v-1}\exp^{-y}dy, (v>0)$$

The shape parameter shows the peakedness meaning that it represents the expected most frequent wind speed.

The scale parameter tells us how the region under study is windy, meaning that it helps in knowing how the distribution of wind speed is expected to spread.

Threshold parameter assist in understanding the expected minimum wind speed value for the region of interest.

3.5. Relative Efficiency

The efficiency test is assisting us to judge the most efficient techniques between the two techniques namely; MLE and MDE fitting method. This test is also used to conclude on the efficient distribution. The efficiency of the distributions was investigated for only the best 2-parameter distribution under the two different fitting technique and for the best 3-p distribution under the two fitting techniques. For both techniques, the best distribution was gamma for 2-p and 3-p analysis. This means that the study used the relative efficiency formula for gamma distribution given by;

$$Gamma = \frac{z_1 q_2^2}{z_2 q_1^2} \quad (53)$$

Where; z is the shape parameter and q is the scale parameter.

The gamma three parameter distribution is given as follows;

$$f(y,z,q,t) = \frac{y\,t^{z-1}}{\Gamma(z/t)q^z}\exp\left[-\left(\frac{y}{q}\right)^t\right](z,y,q,t>0) \quad (51)$$

Where:

q is the scale parameter.

z is shape parameters

t is the thresh-hold parameter

Therefore, the gamma distribution to be fitted will be as follows after inserting the estimated parameters in the equation.

Table 15. Efficiency test for estimation techniques.

Best distribution	Technique	Parameter	Estimate	R. Efficiency
Gamma 2-P	MLE	Shape	2.47634	0.8349
		Scale	1.25991	
	MDE	Shape	2.107526	
		Scale	1.062046	
Gamma 3-P	MLE	Shape	2.071773	0.8733
		Scale	1.120855	
	MDE	Shape	1.864567	
		Scale	0.993702	

From the results in Table 14, the relative efficiencies are 0.8348 and 0.8733 respectively for the best 2-parameter and 3- parameter distributions under MLE and MDE techniques or methods. Because the relative efficiencies are both less than 1, we conclude that MLE is more efficient than MDE and therefore, its shape and scale estimates are unbiased, sufficient and consistent.

There is also need to examine the efficiency between the best two distributions obtained in the study for 2-p and 3-p fitting. Maximum likelihood estimation fitting method obtained gamma distribution as the best in 2-p and 3-p analysis for fitting the wind speed data. Table 16; show the relative efficiency between the best two distributions given by MLE

$$R.E = \frac{Var(Gamma\,(2-P))}{Var(Gamma\,(3-P))} \quad (54)$$

Table 16. Efficiency test for the best 2-P and 3-P distributions.

Distribution	Technique	Parameter	Estimate	R. Efficiency
Gamma 2-P	MLE	Shape	2.47634	1.0571
		Scale	1.25991	
Gamma 3-P	MLE	Shape	2.071773	
		Scale	1.120855	

From Table 16, the relative efficiency value is 1.0571 which is greater than 1, indicating that gamma distribution with 3-p is more efficient compared to gamma distribution with 2-p. This leads us to a conclusion that gamma with 3-p is the best distribution for fitting this wind speed data, it is still the efficient distribution for fitting the wind speed data because its estimated parameters are confirmed to be consistent, sufficient and unbiased.

4. Conclusion

From the analysis, using relative efficiency we can reach a conclusion that MLE is the efficient technique/method for fitting the wind speed data to a probability distribution as compared to MDE technique. This is because from the method of MLE, the study got more precise estimates. Also, the method of MLE gives the best model since it yield smaller AIC and BIC values than the method of MDE.

5. Recommendation

It is recommended that for further work regarding the study of the distributions, MLE method can be used to estimate the parameters since it gives precise estimates.

The study also recommends to researchers, to use other datasets from other parts of the world to examine if the Minimum Distance Estimation techniques can give efficient estimates compared to other fitting techniques like Method of Moments and Least Square Estimation.

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